Trainer pack

Developing the personal maths skills of teachers and assessors

Module 7

Algebra master class

Course information	**Length of session:** approx. 5hr 30 min, **not** including lunch or breaks. Trainers can customise the session to suit the audiences and settings. The session is written to be delivered as one full day, but could equally be delivered as 2 half days.
Audience	**Job roles:** Practitioners who are teaching or supporting adult numeracy on functional skills on embedded and discrete programmes, and who wish to improve their understanding and application of number at Level 3. **Sector / setting:** any part of the learning and skills sector.
Links to other modules	This session forms part of a blended learning programme, aimed at preparing participants for the Level 3 Award in Mathematics for Numeracy Teaching.

Session overview

Activity		Content
1	**Progress review and aims and objectives**	Participants to feed back on progress with course and adopt a solution-focused approach to challenges. Group discussion of incidence of algebraic approaches in private, academic and vocational situations.
2	**Diagnostic assessment - carousel of activities**	Participants engage in a variety of activities exploring algebraic concepts. Through investigation and discussion they build on their awareness of personal development needs.
3	**Solving equations**	Direct teaching of approaches to solving different types of equations – linear, simultaneous and quadratic. It is likely that participants will have widely varying levels of competence and differentiation will be necessary.
4	**Using algebra to solve problems**	Participants engage in group activities to identify mathematics and represent problems algebraically.
5	**Graphical representations**	Participants engage in a variety of activities exploring graphical concepts. Through investigation and discussion they build on their awareness of their development needs.
6	**Drawing and interpreting graphs**	Direct teaching of key underpinning skills associated with the drawing and interpreting of graphs.
7	**Functions, graphs and tables of values**	A group activity designed to consolidate learning and identify areas for further self-study.
8	**Using algebraic and graphical concepts to solve problems**	Participants work in small groups on a variety of differentiated problem solving tasks requiring the application of algebraic and graphical concepts.
9	**Developmental tasks**	Whole group activity to examine the three developmental tasks associated with this master class. Ideas are shared and approaches discussed. Support materials are identified.
10	**Plenary and close**	Revisit assessment criteria for Level 3 course. Revisit aims and objectives. Check on individual progress.

Trainers

Trainer experience or qualifications required

Fully qualified numeracy teacher (Cert Ed / PGCE or equivalent plus numeracy subject specialist qualification); minimum Level 3 mathematics qualification.

Reference material for trainers

Course materials and trainer notes.

Materials used in this pack have been sourced from:

‘1000 problems to enjoy’ www.1000problems.org

	Improving Learning in Mathematics http://tlp.excellencegateway.org.uk/teachingandlearning/downloads/default.aspx#/math Level 3 Advanced Numeracy Preparation Resource pack http://www.excellencegateway.org.uk/node/14310

Participants

Prior knowledge and qualifications	Participants should have a Level 2 maths qualification as a minimum requirement, and some familiarity with basic algebra and trigonometry.
Pre-course activity for participants	All participants should have completed the initial assessment and self-assessment checklist, and have attended the course induction session. Reference is made in the final session of this master class to the self-assessment process participants have undertaken during the initial assessment. Therefore it would be useful for participants to bring along their self-assessment and perhaps also their ILP.

Resources

Resources for reference during the session	As provided or any suitable alternatives chosen by the trainer.
Before the session the trainer needs to:	Prepare all resource materials: The only handout for participants is HO 1: Reflective log – this is available further down in this Trainer pack. There are no PowerPoint slides. Please refer to the Teacher notes below for details of resources and handouts to be prepared for each activity. Note that there are **two** carousel-type sessions: TN 2 and TN 8, where various resources need to be prepared. Suggestions are given, but you may want to replace these with your own ideas or activities that more accurately reflect the learning needs of your particular group of participants. TN 3 activity includes optional extension material (quadratic equations activity) – you will need to supply

this, based on your own experience and the needs of participants.

Note that some resource materials (e.g. R 18) have the solutions as well as the problem. You will need separate them and hand the solutions separately at an appropriate point in the session.

Trainers will need to bring along copies of the Development tasks for algebra, or, if this is possible, show them online.

Notes for trainer

Trainer pack
All handouts and resources are included at the end of this document, for ease of printing

Aims

To revise / widen participants' personal mathematical skills – particularly: algebraic representation, manipulation of formulae, problem solving using formulae, and their impact on mathematical understanding and modelling.

Outcomes

By the end of the session participants will have:

- reflected upon personal maths skills and identified areas for development;
- revised and consolidated algebraic strategies and manipulations; and
- identified and applied appropriate algebraic and graphical concepts in order to solve problems.

Suggested timings are for guidance purposes only. Trainers should adapt the content to reflect the results of initial assessment and to meet the needs and experience levels of the participants.

Session plan

TN – Teacher notes **HO – Handout** **R – Resources**

<table>
<tr><th rowspan="2">Time</th><th rowspan="2">Content</th><th colspan="3">Resources</th></tr>
<tr><th>No.</th><th>Style</th><th>Title</th></tr>
<tr><td>15m

(Total 15m)</td><td>TN 1. Welcome; aims and outcomes

Each participant to give brief account of progress since the Induction workshop and the Number master class.

Brief review of developmental tasks undertaken since the previous session.</td><td></td><td></td><td></td></tr>
<tr><td>45m</td><td>TN 2. Assessment and investigation activity

Explore attitudes to ‘algebra’ - past and present. Explore experience of algebra and thoughts as to where it can be utilised. Part of the purpose of the activity is to explore participants’ understanding and confidence.

Paired activity
Carousel of algebra activities and investigations. These should now be differentiated according to original initial assessment, ILPs and knowledge of individual participants. Trainers may develop their own activities or use / adapt the following resources.
Ensure there are sufficient activities for the group size. Note that some of these resources require preparation (making cards).
• R1: Algebra racetrack
• R 2: Negative indices
• R 3: Expressions and equations
• R 4: Match the sequences
• R 5: Evaluating equations and inequalities
• R 6: Matching algebraic expressions with explanations in words

The trainer should circulate encouraging participants to work together to problem solve and discuss their approaches. Observation and questioning should be</td><td>R 1 - R 6</td><td>Carousel of activities</td><td>(See titles opposite)</td></tr>
</table>

Time	Content	Resources		
		No.	Style	Title
(Total 1h)	used to identify individual needs. Take group feedback and clarify as required. Give time to reflect on what they need to do to improve their progress and make notes on ILP, using **HO 1**.	HO 1	Handout	Reflection sheet
1h	**TN 3. Solving equations** This section will be dependent on the underpinning skills of participants. Differentiate as required and adjust time and activities accordingly. **Linear equations – direct teaching** Work through simple examples of linear equations and more complex examples of linear equations. • Simple linear equation e.g. $3x + 4 = 19$ • More complex e.g. $3(x - 4) = x + 10$ Take feedback on solution processes: expanding brackets, making the unknown the subject of the equation, etc. Refer to processes used in Induction Workshop when forming equations – balance, reverse operations, etc. **Simultaneous equations – paired activity** Give each pair a set of the cards prepared **from R 7** – set one (blue) has the description of steps to solution and set two (green) has the worked example of the step. Give time for participants to match and order the cards. Take feedback from the group. What happens if the terms are not in the same order? What happens if the coefficients of both unknowns are different? **Discussion, consolidation and direct teaching** • Principle of equating the two equations.	R 7	Card match	Simultaneous equations - ordering steps

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<tr><th rowspan="2">Time</th><th rowspan="2">Content</th><th colspan="3">Resources</th></tr>
<tr><th>No.</th><th>Style</th><th>Title</th></tr>
<tr><td>(Total 2h 30m)</td><td>What you do to one you do to the other in order to isolate single term and so solve?
Review more complex steps – different coefficients, etc.
Work through examples together (see HO 8 which also has the solutions).
Using simultaneous equations to solve problems.

Quadratic equations – direct teaching
Work through simple examples by factorising. Be prepared to have extension material for participants to solve by completing the square and by formula. The resource sheet R 9 can be used for whole group teaching or for differentiated individual work.
$(x - 3)(x - 4) = 0$
$x^2 - 6x + 5 = 0$
Give time to reflect on what they need to do to improve their progress and make notes on ILP, using HO 1.</td><td>R 8

R 9

HO 1</td><td>Question sheet

Information and question sheet

Handout</td><td>Simultaneous equations - problem solving

Solving quadratic equations

Reflection sheet</td></tr>
<tr><td>20m

(Total 2h 50m)</td><td>TN 4. Using algebra to solve problems

Differentiate into two groups and give each a set of problems in words, using R 10.
Step 1 – write the word problem as an equation. Hand out solution equations R 11.
Step 2 – match solution equations with ones from Step 1.
Discuss differences – do they match? What additional processes have been undertaken? What other algebraic conventions have been used?

Extension
Solve the equations using the processes outlined. Note: do all equations have a solution? Discuss implications.

Give time to reflect on what they need to do to improve their progress and make notes on ILP, using HO 1.</td><td>R 10

R 11

HO 1</td><td>Card match

Card match

Handout</td><td>Forming equations – words

Forming equations – equations

Reflection sheet</td></tr>
</table>

Time	Content	Resources		
		No.	Style	Title
20m *(Total 3h 10m)*	**TN 5. Graphical representation** **Paired / small group activity** Work through carousel activities – explore and discuss findings. Make notes for feedback to whole group. • Matching linear and quadratic equations to graphs (LLU+ materials) **R 12** • Real life graphs (LLU+ materials) **R 13** • Speed distance time graphs (functional skills materials) **R 14** Share findings and draw out any misconceptions and inaccuracies. Give time to reflect on what they need to do to improve their progress and make notes on ILP, using **HO 1**.	R 12 R 13 R 14 HO 1	Card match Card match Investigation Handout	Matching linear and quadratic graphs with equations Real life graphs Speed, distance and time Reflection sheet
45m *(Total 3h 55m)*	**TN 6. Drawing and interpreting graphs** **Direct teaching** Revise and establish properties and essentials of graphs. Encourage all to contribute and draw out points related to: • Scale and axes • Gradient • Intercept • Linear versus non-linear relationships • Interpretation • Plotting of points – co-ordinate pairs • Preparation of tables of values • Equation of straight line Record for all to see. Hand out **R 15** and support participants to work through the questions. Note that the graphs referred to in Question 1 are those in **R 12.** Encourage pairings where one may support and guide the other. Take feedback on findings. Give time to reflect on what they need to do to improve their progress and make notes on ILP, using **HO 1**.	R 15 HO 1	Investigation Handout	Linear graph activities Reflection sheet

Time	Content	Resources		
		No.	**Style**	**Title**
20m *(Total 4h 15m)*	**TN 7. Functions, graphs and tables of values** **Purpose of this activity**: this activity can be used to assess progress during the day, to consolidate what has been learnt and to signpost to further study. Prepare the four sets of cards (**R 16**: Card set A: Equations; Card set B: Words; Card set C: Graphs; Card set D: Tables of values) in advance. In the process of matching words to equations to graphs to tables, participants will be required to: • interpret algebraic symbols • manipulate equations • distinguish between linear and quadratic functions • substitute values into equations • read graphs • identify the effect of doubling and squaring positive and negative numbers. **In small groups** • Hand out word and equation cards (**R 16**, sets A and B) and match them (indicate that it is not a 1:1 relationship – circulate and ask participants to justify answers • Hand out graph cards (Set C) and match to existing cards– circulate and ask participants to justify answers • Hand out tables cards (Set D) to be matched with the others. Take feedback. Give time to reflect on what they need to do to improve their progress and make notes on ILP, using **HO 1**.	R 16 HO 1	Card match Handout	Matching functions, graphs and tables Reflection sheet
45m *(Total 5h)*	**TN 8. Using algebraic and graphical concepts to solve problems** Work in small groups to solve problems: • **R 17**: Linear or non-linear? • **R 18**: Which charging option?	R 17 - R 20	Resources	Problem questions (see opposite for

<table>
<tr><th rowspan="2">Time</th><th rowspan="2">Content</th><th colspan="3">Resources</th></tr>
<tr><th>No.</th><th>Style</th><th>Title</th></tr>
<tr><td></td><td>R 19:The swimming race
R 20: Which is hotter?
Give time to reflect on what they need to do to improve their progress and make notes on ILP, using HO 1.</td><td></td><td></td><td>details)</td></tr>
<tr><td>15m

(Total 5h 15m)</td><td>TN 9. Developmental tasks

Whole group activity to examine the three developmental tasks associated with this master class. Ideas are shared and approaches discussed.

Support materials, for the tasks, or for personal study, are identified.

Give time to reflect on what they need to do to improve their progress and make notes on ILP, using HO 1.</td><td></td><td></td><td></td></tr>
<tr><td>15m

(Total 5h 30m)</td><td>10. Plenary and close

Revisit aims and outcomes.

Review self-assessment checklist (from the initial assessment); make any necessary adjustments to the ILP.</td><td></td><td></td><td></td></tr>
</table>

Algebra master class – resources

Note that the **resources** need to be prepared by the trainer and include several which are sets of cards. There is only one **handout** for this session. Note also that some of the resources include solutions – you might want to distribute these only at the appropriate time.

Handouts

HO 1: Reflection sheet

Resources

R 1: Algebra racetrack
R 2: Negative indices
R 3: Expressions and equations (plus solutions)
R 4: Match the sequences (note that the layout of the resource gives the correct responses)
R 5: Evaluating equations and inequalities
R 6: Matching expressions and descriptions
R 7: Simultaneous equations
R 8: Problems involving simultaneous equations (includes solutions)
R 9: Solving quadratic equations
R 10: Forming equations – words
R 11: Forming equations – equations
R 12: Matching linear and quadratic graphs with equations
R 13: Real life graphs
R 14: Speed, distance and time
R 15: Linear equations activities
R 16: Matching functions, graphs and tables (Card sets A, B, C and D)
R 17: Linear or non-linear? (includes solutions)
R 18: Which charging option? (includes solution)
R 19: The swimming race
R 20: Which is hotter? (includes solution)

HO 1: Reflection sheet

Use this sheet to record your own reflections and development needs.

Progress review and aims and objectives	
Diagnostic assessment – carousel of activities	
Solving equations	
Using algebra to solve problems	
Graphical representations	
Drawing and Interpreting graphs	
Functions, graphs and tables of values	
Developmental tasks	
Plenary and close	

R 1: Algebra racetrack

<table>
<tr><td>Start
Finish</td><td>$2n - 4$</td><td>$n^2 + 2$</td><td>$2(n + 1)$</td><td>$3(n - 4)$</td></tr>
<tr><td>$4 - n$</td><td colspan="3" rowspan="5">Rules

Each player will need a dice and a counter.
Place your counter on the Start / Finish square.
Decide how many times you have to go round the board before you finish.
Roll the dice. Use the score on the dice together with the instruction on the square to work out how many spaces to move. (For example, a dice score of 4 with the instruction ‘Add 3’ means you can move 7 spaces.)
Move your counter forwards or backwards, according to your total score.

Pass the dice to the next player.</td><td>$n + 3$</td></tr>
<tr><td>$2n$</td><td>$n + 2$</td></tr>
<tr><td>$n - 2$</td><td>$2(n - 3)$</td></tr>
<tr><td>$7 - 2n$</td><td>$n - 3$</td></tr>
<tr><td>$2(n + 1)$</td><td>$10 - n$</td></tr>
<tr><td>$3(n - 3)$</td><td>$n + 5$</td><td>$n - 1$</td><td>$2n$</td><td>$8 - (n + 2)$</td></tr>
</table>

R 2: Negative indices

The cards can be cut up and used to match equivalent forms.

In general: $$x^{-n} = \frac{1}{x^n}$$

(It can be helpful to read negative indices as ‘divide by’ the positive power)

$\frac{1}{64}$	8^{-2}	$(\frac{1}{8})^2$
$\frac{1}{4^3}$	4^{-3}	$\frac{1}{64}$
$\frac{1}{2^4}$	2^{-4}	$\frac{1}{16}$
¼	4^{-1}	½
⅛	8^{-1}	⅓
2^{-1}	3^{-1}	

Taken from: LSIS Advanced Numeracy Preparation Resource Pack (for Level 3 preparation programmes for entry to numeracy teacher training)
http://archive.excellencegateway.org.uk/page.aspx?o=295882

R 3: Expressions and equations

For each statement decide whether ‘x + 5’ is:

Definitely true / Definitely false / Possibly true

(The first one has been completed)

Repeat for $x^2 + 5$

Statement	$x + 5$	$x^2 + 5$
$x + 5$ / $x^2 + 5$ is an Equation	Definitely false	
$x + 5$ / $x^2 + 5$ is an Expression		
$x + 5$ / $x^2 + 5$ Equals (-4)		
$x + 5$ / $x^2 + 5$ is Quadratic		
$x + 5$ / $x^2 + 5$ is Cubic		
$x + 5$ / $x^2 + 5$ Equals zero		
$x + 5$ / $x^2 + 5$ Equals 105		
$x + 5$ / $x^2 + 5$ is a A term		
$x + 5$ / $x^2 + 5$ is Linear		

Expressions and equations – solutions

Statement	$x + 5$	$x^2 + 5$
Equation	Definitely false	Definitely false
Expression	Definitely true	Definitely true
Equals (-4)	Possibly true	Possibly true*
Quadratic	Definitely false	Definitely true
Cubic	Definitely false	Definitely false
Equals zero	Possibly true	Possibly true*
Equals 105	Possibly true	Possibly true
A term	Definitely false	Definitely false
Linear	Definitely true	Definitely false

Taken from: LSIS Advanced Numeracy Preparation Resource Pack (for Level 3 preparation programmes for entry to numeracy teacher training)
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R 4: Match the sequences

Cut out the cards and ask participants to match them.

Triangular numbers	1, 3, 6, 10, 15 …..
Square numbers	1, 4, 9, 16 …..
Cubic numbers	1, 8, 27, 64 ……
Fibonacci sequence	1, 1, 2, 3, 5, 8 …….
Odd numbers	1, 3, 5, 7, 9 ……
Even numbers	2, 4, 6, 8, 10 …..
Prime numbers	2, 3, 5, 7, 11…….
Powers of 10	1, 10, 100, 1000, 10000 ……
Powers of 2	1, 2, 4, 8, 16 ……..
A convergent series	1, $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$ ……..
Multiples of one eighth	0.125, 0.25, 0.375, 0.5, 0.625 …

R 5: Evaluating equations and inequalities

Review each equation / inequality and decide whether it is Always True, Sometimes True or Never True. Give examples to support your decision. (Give yourself examples to try to come to a decision!)

Taken from LSIS Improving Learning in Mathematics (Sessions A4):
http://tlp.excellencegateway.org.uk/pdf/mat_imp_02.pdf

1 $n + 5 = 11$	2 $q + 2 = q + 16$
3 $2n + 3 = 3 + 2n$	4 $2t - 3 = 3 - 2t$
5 $3 + 2y = 5y$	6 $p + 12 = s + 12$
7 $4p > 9 + p$	8 $n + 5 < 20$
9 $2(x + 3) = 2x + 3$	10 $2(3 + s) = 6 + 2s$
11 $x^2 > 4$	12 $x^2 = 5x$
13 $x^2 > x$	14 $9x^2 = (3x)^2$

R 6: Matching expressions and descriptions

Taken from LSIS Improving Learning in Mathematics (Sessions A1):
http://tlp.excellencegateway.org.uk/pdf/mat_imp_02.pdf

E1 $\dfrac{n+6}{2}$	E2 $3n^2$
E3 $2n+12$	E4 $2n+6$
E5 $2(n+3)$	E6 $\dfrac{n}{2}+6$
E7 $(3n)^2$	E8 $(n+6)^2$
E9 $n^2+12n+36$	E10 $\dfrac{n}{2}+3$
E11 n^2+6	E12 n^2+6^2
E13	E14

R 7: Simultaneous equations

Ordering steps

Label equations (1) and (2).	$3x + 2y = 12$ (1) $5x - 2y = 4$ (2)
Check to see if there is the same number of xs or ys in both equations.	**There are the same number of ys**
Decide whether to add or subtract equations to eliminate x or y.	**Add equations as the y signs are different**
Add the equations together.	$3x + 2y = 12$ (1) $\underline{5x - 2y = 4\ }$ (2) $8x = 16$
Solve the equation to find the value of one of the letters.	$x = 2$
Substitute the value you have found into one of the original equations.	$3x + 2y = 12$ (1) $6 + 2y = 12$
Solve the equation to find the value of the remaining letter.	$6 + 2y = 12$ $2y = 6$ $y = 3$
Check solutions work by substituting both values into the other original equation.	$x = 2$; $y = 3$ $5x - 2y = 4$ (2) $10 – 6 = 4$

Taken from: LSIS Advanced Numeracy Preparation Resource Pack (for Level 3 preparation programmes for entry to numeracy teacher training)
http://archive.excellencegateway.org.uk/page.aspx?o=295882

R 8: Problems involving simultaneous equations

Solve the following problems using simultaneous equations.

In each case represent each unknown with a letter. Form two separate equations and then follow the usual steps to solve them simultaneously.

1. In an election Mrs White was elected with a majority of 346 over Mr Brown. The total number of votes cast was 1418. How many votes did each candidate receive?

2. On a package holiday, two adults and one child can go for £1190. Similarly, the fare for one adult and three children is £1320. How much does it cost for an adult and a child?

3. The staff room coffee machine takes 20p or 50p coins. When emptied, it was found to contain 36 coins totalling £10.50 in value. How many of each sort of coin did the machine contain?

Solutions

Question 1

Mrs White: 882

Mr Brown: 536

Question 2

Adults: £450

Children: £290

Question 3

20p: 25 coins

50p: 11 coins

R 9: Solving quadratic equations

Quadratic equations

Quadratic equations can be written in the form:

$$ax^2 + bx + c = 0$$

You may need to rearrange the equation so that it looks like this and of course the signs may be different. The main thing is that there is an x^2 term but nothing of a higher power than that.

Quadratic equations have up to two solutions (they may have one and they may have none).

They can be solved in a number of ways – we will consider by factorising and graphically. (Other ways are by using the formula and completing the square – you will need to look at the extension materials on the web site for these.)

Factorising

Algebraic expressions have factors just like numbers do. For example the factors of 3xy are 3, x, y, 3xy and 1 (give yourself another example and work out the factors).

Some quadratics can be factorised because they have whole number solutions which are relatively easy to see.

It is best to revise multiplying brackets first because it is easier than the reverse!

Take: $(x + 3)(x + 4)$

If we multiply this out we have $x(x + 4) + 3(x + 4)$

This equals $x^2 + 4x + 3x + 12$

Simplified this equals $x^2 + 7x + 12$

Try these:

1. $(x + 1)(x + 2)$
2. $(x + 3)(x + 5)$
3. $(x - 1)(x + 4)$
4. $(2x + 1)(3x + 5)$

So when we are factorising quadratics we are looking for two numbers which multiply to give the constant term and add up to give the coefficient of the *x* term:

$$(x + a)(x + b) = x^2 + (a + b)x + ab$$

Example:

To factorise $x^2 + 5x + 4$ we need to find two numbers that multiply to give 4 and add up to give 5. Factors of 4 are 1, 2, 4 so we choose 4 and 1 as their product is 4 and their sum is 5.
We then write this as: $(x + 4)(x + 1)$

Now do these:

1. $x^2 + 16x + 64$
2. $x^2 + 11x + 24$
3. $x^2 + 5x + 4$
4. $x^2 + 7x + 10$

Once we introduce negative numbers we have to think carefully about the signs. Try these:

1. $x^2 - 9x + 18$
2. $x^2 + 3x - 18$
3. $x^2 + 5x - 14$
4. $x^2 - 4x - 21$

Try these – multiply out your brackets afterwards to check:

1. $3x^2 + 5x + 2$
2. $9x^2 + 6x + 1$

Now you are ready to solve some quadratic equations:

If we know that: $x^2 + 8x + 12 = 0$

Then $(x + 6)(x + 2) = 0$

So either $(x + 6) = 0$ or $(x + 2) = 0$ (Why?)

So $\boldsymbol{x = -6}$ **or** $\boldsymbol{x = -2}$

1. $x^2 + 4x + 3 = 0$
2. $x^2 + 10x + 16 = 0$
3. $x^2 + 8x + 12 = 0$
4. $x^2 + 6x - 7 = 0$
5. $x^2 - 5x - 6 = 0$
6. $7x^2 + 8x + 1 = 0$

One side of a rectangle is 10cm longer than the other. If the area of the rectangle is $56cm^2$, form a quadratic equation and find the length of the two sides.

R 10: Forming equations – words

The sum of two consecutive even numbers is 36.	The length of a rectangle is 8cm longer than the width. The perimeter is 36cm.
A bucket of water weighs 36kg. The water weighs three times as much as the bucket.	A length of rope of length 36m is cut into three sections. The first section is x metres long, the second section is half this length, and the third section is 8m shorter than the first section.
Sian says that she is three times as old as her son Glynn. Glynn says that Sian is 36 years older than he is. They are both right.	Bob bought 8 cans of beer for himself and his friends. He bought x cans for himself at £2 each and the remainder (for his guests) at £1 each. He spent £36 altogether.
Dee, Mary and Nicola go out together. Dee takes £x spending money. Mary takes three times as much as Dee. Nicola takes £8 more than Dee. They take £36 altogether.	

Taken from: LSIS Advanced Numeracy Preparation Resource Pack (for Level 3 preparation programmes for entry to numeracy teacher training)
http://archive.excellencegateway.org.uk/page.aspx?o=295882

R 11: Forming equations – equations

$x + (x + 2) = 36$	$3x = x + 36$
$x + \frac{x}{2} + (x - 8) = 36$	$x + \frac{x}{2} = 36$
$2x + (8 - x) = 36$	$2x + 2(x + 8) = 36$
$4x + x + 8 = 36$	

Taken from: LSIS Advanced Numeracy Preparation Resource Pack (for Level 3 preparation programmes for entry to numeracy teacher training)
http://archive.excellencegateway.org.uk/page.aspx?o=295882

R 12: Matching linear and quadratic graphs with equations

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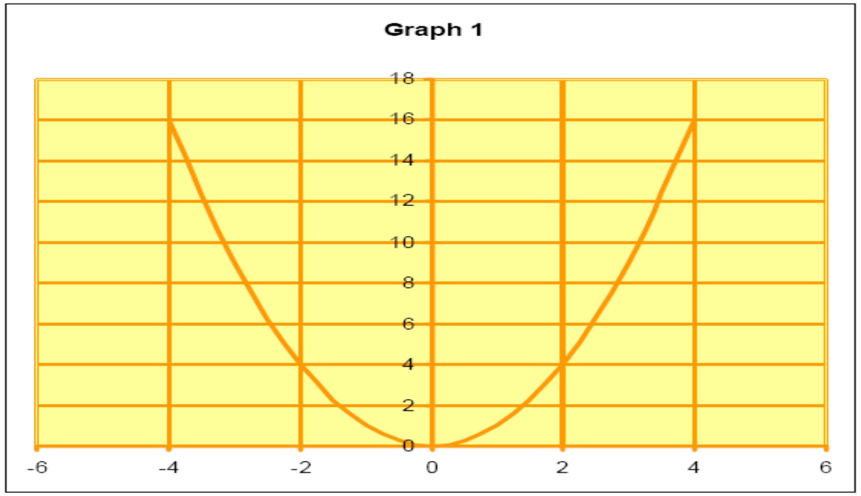
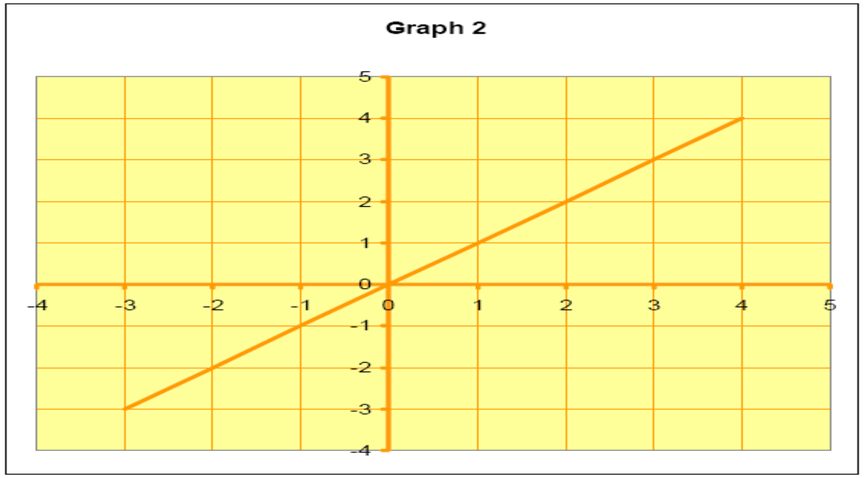
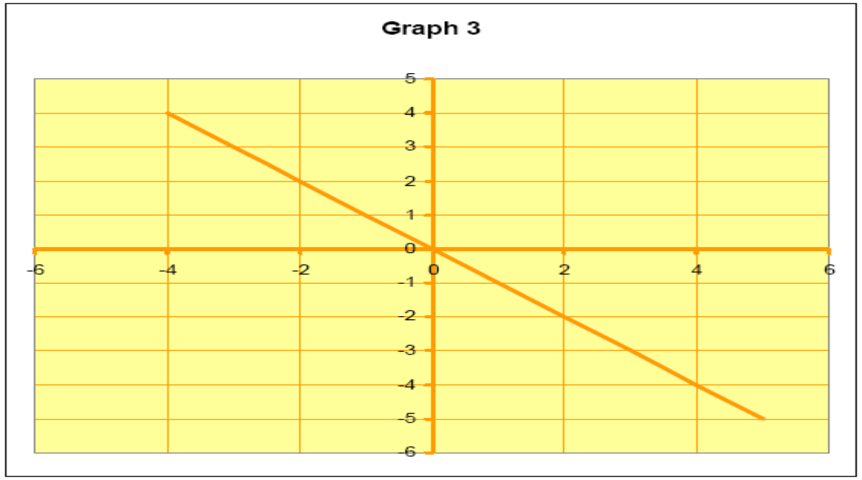
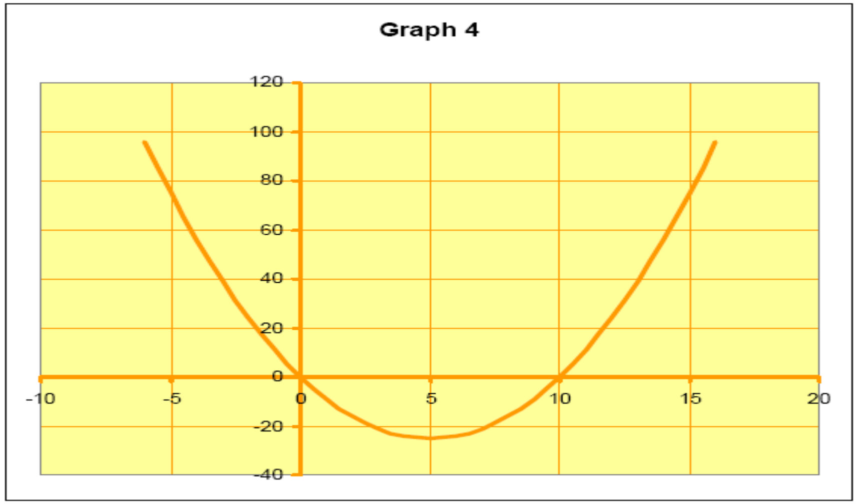
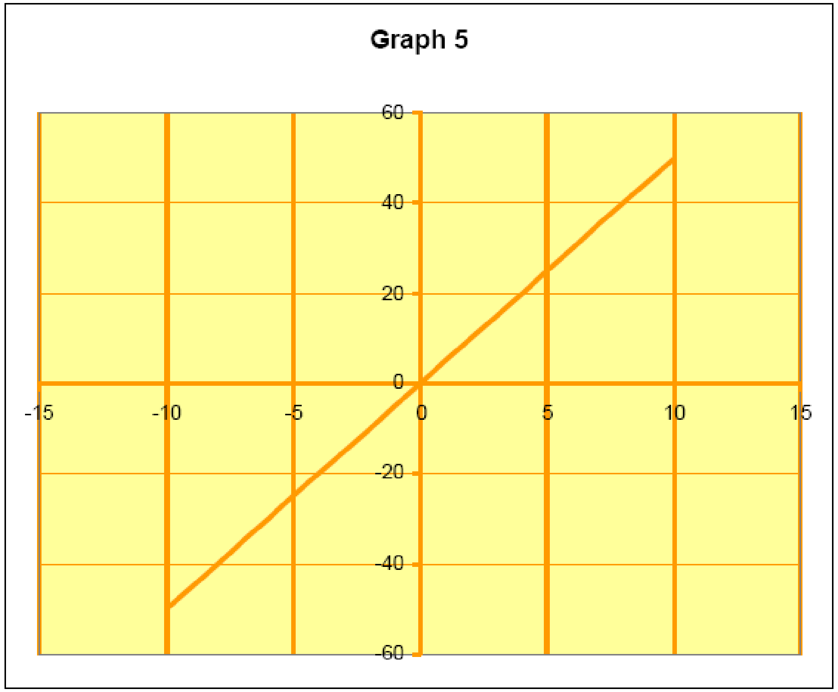
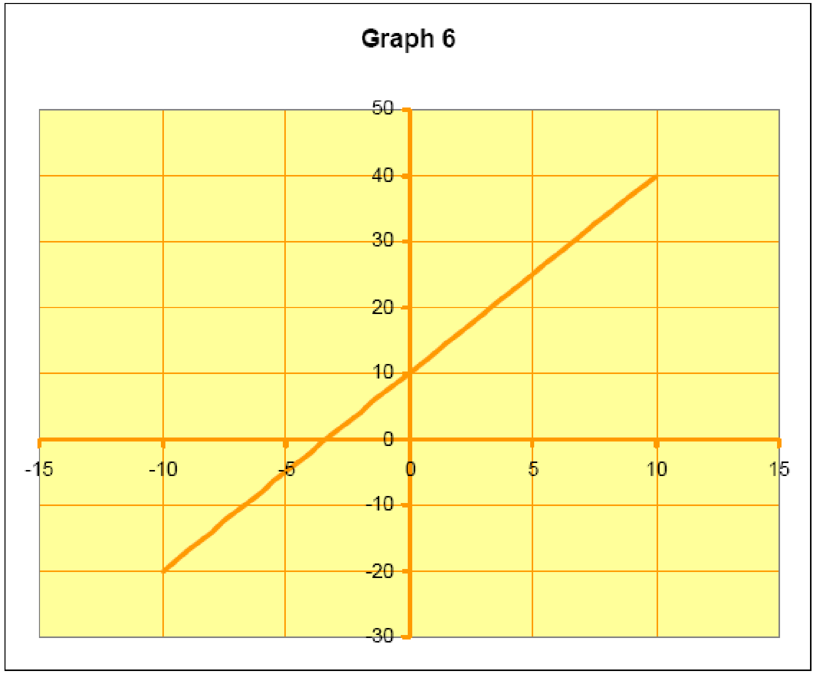








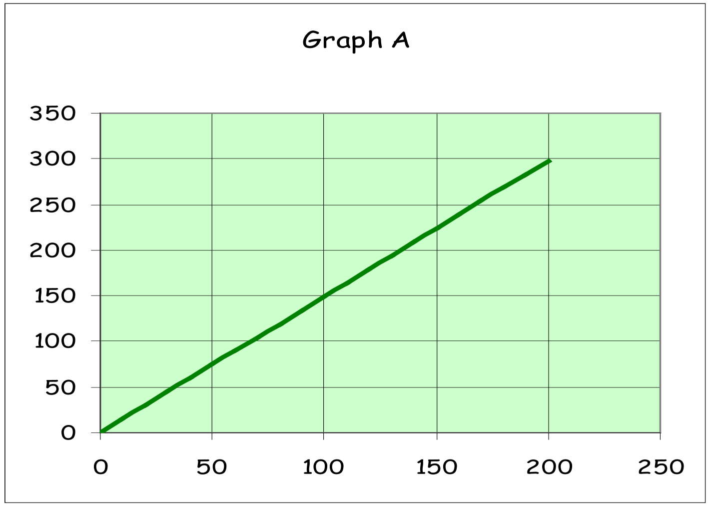


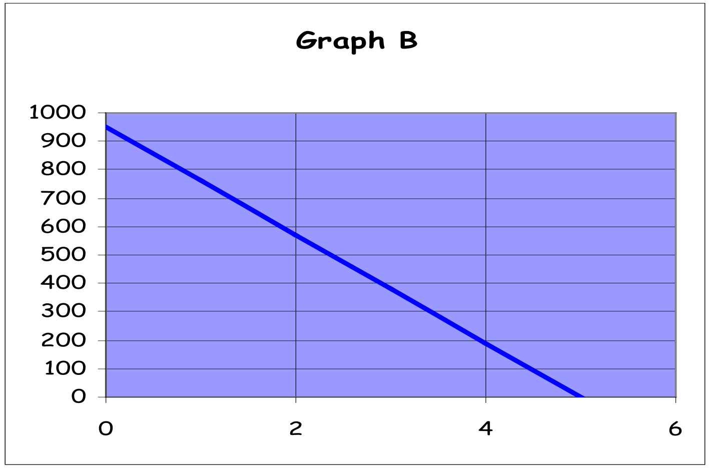


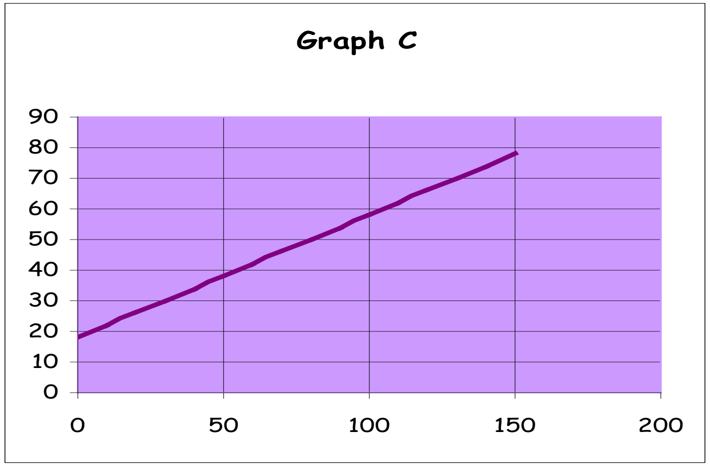
$y = 3x + 10$	$y = 5x$	$y = x^2 - 10x$
$y = -x$	$y = x^2$	$y = x$

R 13: Real life graphs

Taken from: LSIS Advanced Numeracy Preparation Resource Pack (for Level 3 preparation programmes for entry to numeracy teacher training) http://archive.excellencegateway.org.uk/page.aspx?o=295882







$y = 1.49x$

$y = 950 - 190x$

$y = 0.4x + 18$

This graph shows the cost of printing wedding invitation cards. It gives the cost against the number of cards.

How much water had been let out of the bath after 2½ minutes?

This graph shows water being let out of a bath. It gives the capacity in litres against the time.

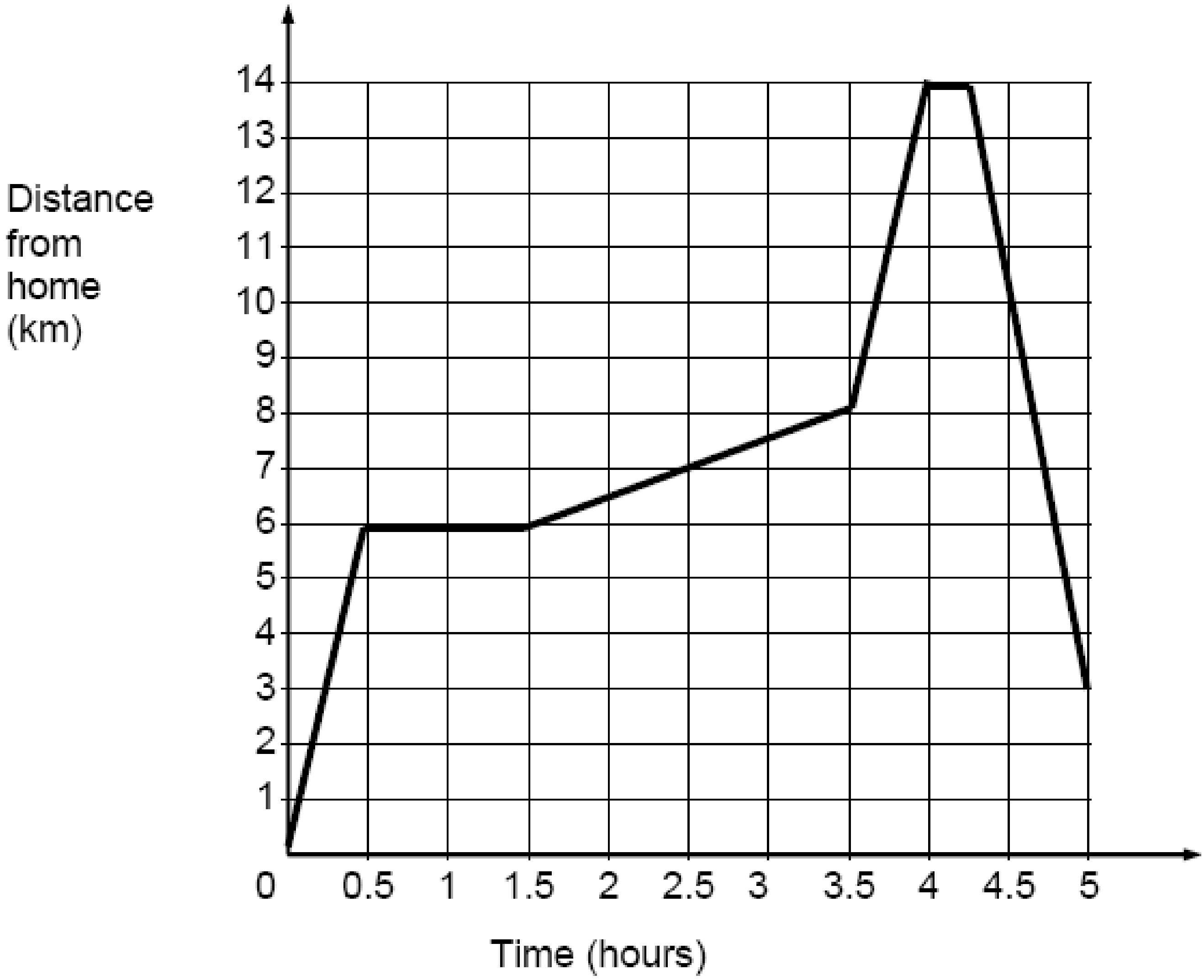
I need 100 cards for my wedding. How much will this cost?

This graph shows the conversion of Euros against pounds.

How much is £15.00 in Euros?

R 14: Speed, distance and time

Graphs that tell stories



Andrew sets off on a cycle ride as shown in the graph above. What is his average speed for the whole journey?

What is his speed for each section of the journey?

Taken from Teaching and Learning Functional Mathematics - Resources to support the pilot of functional skills: http://archive.excellencegateway.org.uk/page.aspx?o=126261

Extending the story

- **Is the graph realistic? If not, why not? What assumptions have been made? How could you make it more realistic?**

- **Where did he fall off his bike?**
- **What happened at the end? Why do you think he did not go home?**
- **When was he riding at his fastest?**
- **Why do you think he was going fast on this part of the journey?**
- **What could have been happening after 3½ hours of the ride?**
- **Describe the whole journey in words.**
- **How would the graph have been different if he had got a puncture somewhere and had to walk the rest of the way?**
- **How could you change the graph so that his average speed is 4 km h^{-1}? Or 6 km h^{-1}?**

R 15: Linear equations activities

1. Draw families of lines which are parallel to the ones in graphs 2, 3, 5 and 6 of the carousel activity, in R 12.

2. Note down points below that you have established about straight line graphs. (Hint – these may be to do with gradient, intercept on the y axis, positive and negative gradients, etc,)

   a)

   b)

   c)

3. Work in pairs to make up simple linear equations. Swap your equation with your partner who must then draw the line. Note down what processes and approaches you might use.

   a)

   b)

   c)

R 16: Matching functions, graphs and tables

Taken from LSIS Improving Learning in Mathematics (Sessions A7): http://tlp.excellencegateway.org.uk/pdf/mat_imp_02.pdf

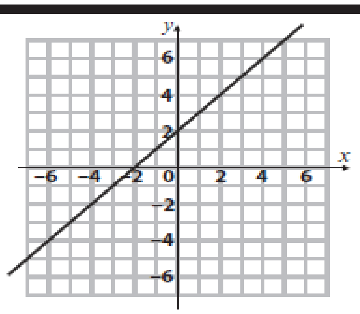
Card set A: Equations

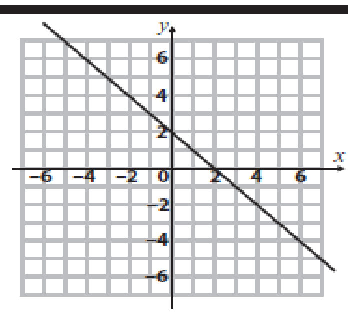
$y = x^2$	$y = 2x$
$y^2 = x$	$y = x + 2$
$2y = x$	$y = 2$
$y = x - 2$	$xy = 2$
$y = \pm\sqrt{x}$	$x + y = 2$
$y = \frac{2}{x}$	$y = \frac{x}{2}$
$y = -x + 2$	$x = \pm\sqrt{y}$

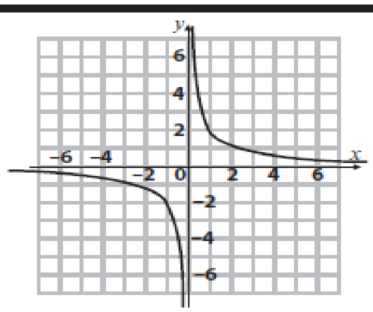
Card set B: Words

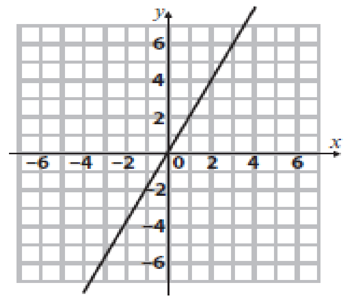
y is one half the size of x	x added to y is equal to 2
y is 2 more than x	x multiplied by y is equal to 2
y is 2 less than x	y is double the size of x
y is always equal to 2	x is the same as y multiplied by y
y is the same as 2 divided by x	y is the same as x multiplied by x
x is the square root of y	y is the same as x divided by 2

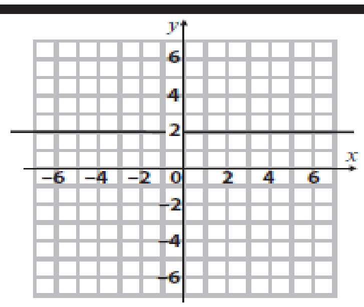
Card set C: Graphs

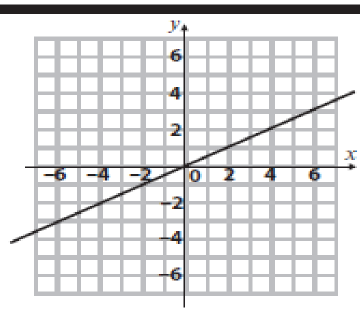


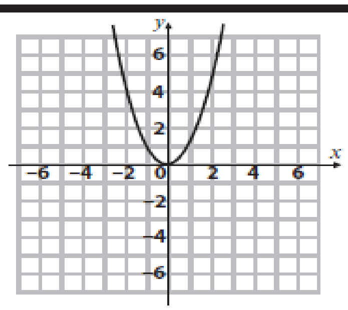


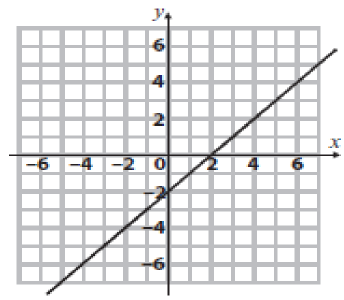


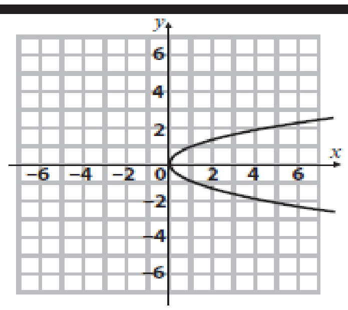












Card set D: Tables of values

x	–2	–1	0	1	2	3
y	–4	–3	–2	–1	0	1

x	–2	–1	0	1	2	3
y	4	3	2	1	0	–1

x	–2	–1	0	1	2	3
y	–1	–0.5	0	0.5	1	1.5

x	–2	–1	0	1	2	3
y	–4	–2	0	2	4	6

x	0	1	4	9	16
y	0	±1	±2	±3	±4

x	–2	–1	0	1	2	3
y	0	1	2	3	4	5

x	–2	–1	0	1	2	3
y	4	1	0	1	4	9

x	–2	–1	0	1	2	4
y	–1	–2	±∞	2	1	0.5

x	–2	–1	0	1	2	3
y	2	2	2	2	2	2

R 17: Linear or non-linear?

LSIS Improving Learning in Mathematics (Sessions A5):
http://tlp.excellencegateway.org.uk/pdf/mat_imp_02.pdf

Plot the sets of points on a graph to determine whether there is a linear relationship or not.

Taxi hire

No. of miles	0	1	2	3	4	5	6
Cost of hiring taxi (£)	5	6.2	7.4	8.6	9.8	11	12.2

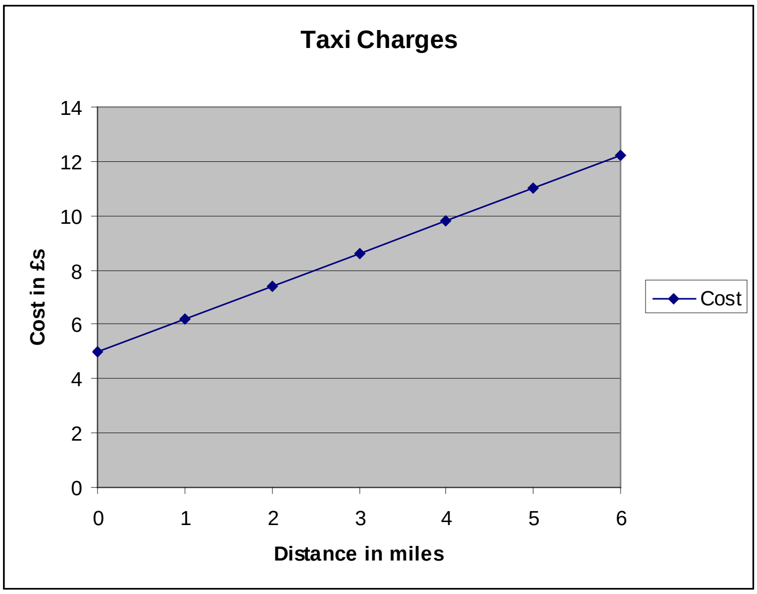
a) Is the relationship linear?
b) Do you think it is likely to be the case in real life?
c) What other factors may affect the relationship?
d) Can you write an equation for the relationship between the cost and the number of miles travelled?
e) At the same rate what would be the cost of travelling 25 miles?

Area of a square

Length of side of square (cm)	1.2	1.7	2.1	2.5	3	3.6
Area of square (cm^2)	1.44	2.89	4.41	6.25	9	12.96

a) Is the relationship linear?
b) Are there other factors which could affect the relationship?
c) How does the relationship fit with what you know already?
d) Can you find an equation which fits your graph?

Solutions



a) Yes, it is linear.

b) Shorter journeys are often more expensive so it is not likely to be linear in real life.

c) Number of people, cases, length of journey time, etc.

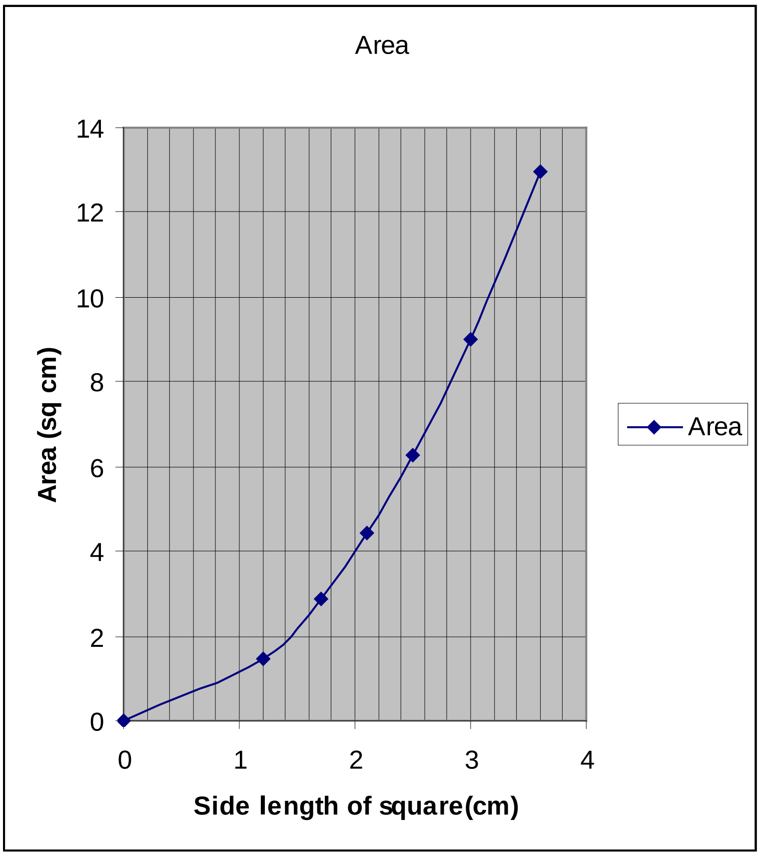
d) **C = 1.2m + 5** (Where C is cost in £s and m is number of miles.)

To travel 25 miles

C = (1.2 x 25) + 5

C = 35

Cost of travelling 25 miles is **£35**.



a) No it is not a linear relationship.

b) No it is a fixed relationship.

c) It is linked to the ratio of similar shapes (all squares are similar to each other): 'the ratio of the area of similar shapes is the square of the ratio of their side length'.

d) The equation is:

$$A = s^2$$

Where A is the area of the square and s is the side length.

R 18: Which charging option? (oil charges)

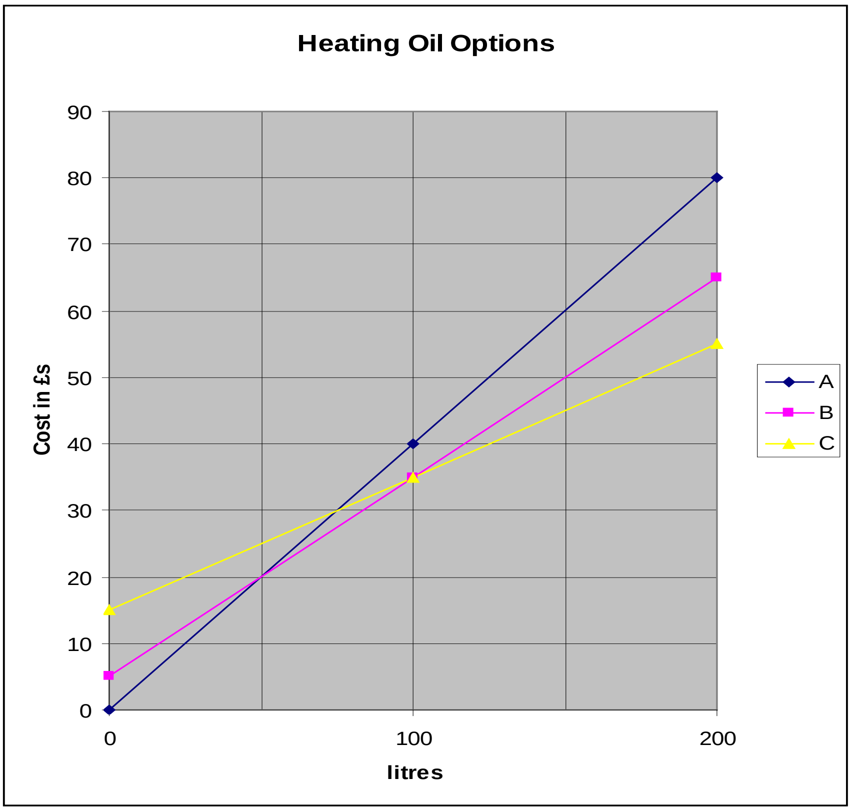
The problem

Central heating oil may be bought in one of three ways:

- Option A - 40p per litre
- Option B - Fixed charge of £5 and 30p per litre
- Option C - Fixed charge of £15 and 20p per litre

Use a graphical solution to determine which option is cheapest over a range of 0 to 200 litres.

Solution



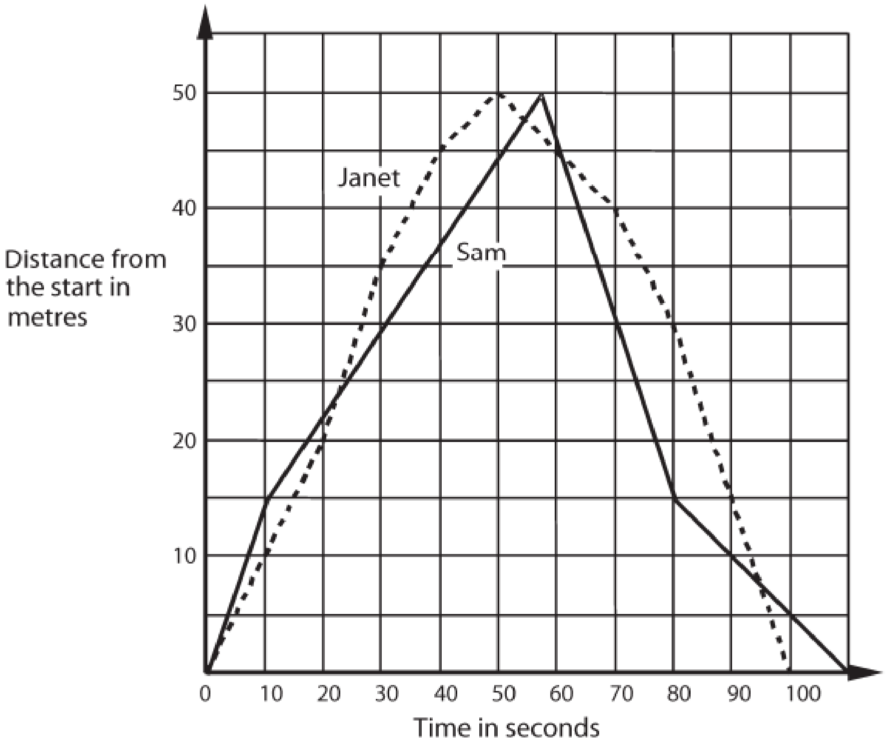
Less than 50 litres – Option A is cheaper.

50 – 100 litres Option B is cheaper

Over 100litres Option C is cheaper

R 19: The swimming race

The following graph describes a swimming race.



The following is a commentary on the race.
Read the commentary and discuss its accuracy.
Justify your opinions by referring to the graph.
Rewrite any parts that you decide are inaccurate.

Sam goes quickly into the lead. He is swimming at 15 metres per second. Janet is swimming at only 10 metres per second. After 22 seconds, Janet overtakes Sam. Janet swims more quickly than Sam from 25 seconds until she turns at 50 seconds. Sam overtakes Janet after 55 seconds, but she catches up again, 5 seconds later. Janet is in the lead until right near the end. Sam swims at a steady 30 metres per second after the turn, until 80 seconds, while Janet is gradually slowing down. Sam wins by 10 seconds.

What other information could be obtained from the graph?

Consider other questions you could ask to prompt someone's interpretation of the graph and associated issues.

R 20: Which is hotter?

The problem

Comparisons between the same attribute measured on different scales can be tricky. For example is 43°C in the Arizona desert hotter than 110°F in Kos? Or who is the taller – Arthur at 6ft 3ins or Jack at 190cm? The level of accuracy required is also an important factor to consider and it is often necessary to state the limits of accuracy.

You can convert from Celsius to Fahrenheit temperatures using the exact, but more complex, formula: $F = \frac{9}{5}C + 32$

or the quick, but approximate, formula: $F = 2C + 30$.

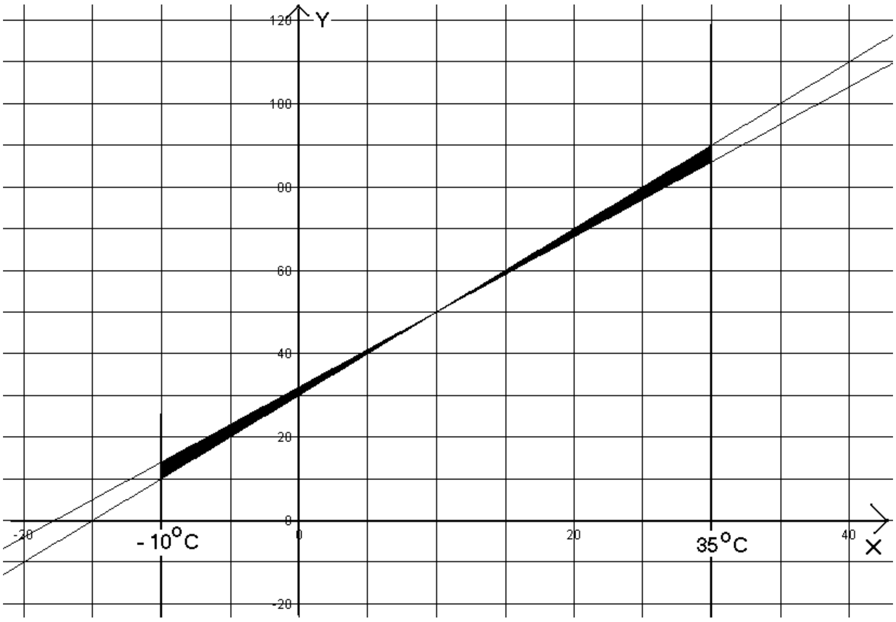
Use a graphical approach to find solutions to the following:

1. Does the quick formula ever give *exactly* the right answer?

2. For what range of Celsius temperatures does the quick formula give an answer that's within 5°F of the correct value?

3. Is the quick formula good enough for our British climate?

Solutions



Equal at 10º C.

Quick formula accurate within 5º C from -10º C to 35º C

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