



Trainer pack

Developing functional mathematics with vocational learners

Module 12b: Number concepts and skills

Course information	Length of session: 3 – 3.5 hours, depending on activities required by participants. Trainers can customise, shorten and lengthen the session to suit the audiences and settings.
Audience	Job roles: Practitioners who are vocational specialists and teaching or supporting the development of numeracy skills or functional mathematics up to and including Level 2. Sector/setting: Vocational learning
Links to other modules	Number concepts and skills (the second of three modules in the collection: <i>Developing functional mathematics with vocational learners</i>) is a module that can be undertaken independently or can be undertaken as part of a group of three CPD modules. The module reinforces the aims and challenges of functional mathematics and the development of conceptual understanding, which is also relevant to Modules 12a: Handling data and Module 12c: Common measures, shape and space.. Developing numeracy with vocational learners: 12a: Handling data 12b: Number concepts and skills 12c: Common measures, shape and space Each module is linked to one aspect of the adult numeracy core curriculum and uses examples and discusses issues concerning how curriculum content might be applied to the needs of vocational learners studying functional mathematics.

Aims

To enable a conceptual understanding of number to be developed with learners in vocational contexts

Outcomes

By the end of the session participants will have:

- identified some techniques and strategies used in calculating and how these can build towards a conceptual understanding of number;
- experienced a range of maths learning activities and have adapted one (or more) to suit relevant vocational contexts;
- reviewed techniques that expose common learner errors and misconceptions in number and identified how to use these to progress learning and understanding; and
- identified challenges to learners presented by functional mathematics accreditation and explored how these may be overcome.

Module overview

Activity		Content
1	Starter: Equivalence cards	This starter activity uses commonly occurring fractions to explore the concept of alternative forms of representation.
2	Introductions	Outline of the session with aims and objectives. Participants introduce themselves.
3	Functional maths in vocational contexts	Participants identify naturally-occurring maths in their own vocational context.
4	Calculating skills related to whole number	This section enables participants to experience the wide range of approaches to carrying out the four operations.
5	Developing conceptual understanding	Participants develop their own conceptual understanding of maths through a variety of activities which they can use, or adapt for use, with their learners.
6	Exploration of common errors	Examining how the exploration of errors and misconceptions can lead to a greater understanding of underlying mathematical concepts.
7	Proportional reasoning	An opportunity to see that enabling learners to work through a process with simple numbers will help them identify which operation to use when the numbers are more complex.
8	Summary and next steps	Reflecting on what has been learned and what can be put into practice.

Note for trainers

Timings are quite tight for many of the activities and trainers may need to adjust activities during the CPD session, depending on the level of knowledge and expertise of participants. Ideally trainers will have the opportunity to explore these well before the session. The planning below is based on participants having a reasonable working knowledge of functional skills; this is not a functional skills awareness session but rather a session which focuses on the more practical issues of interpreting these requirements within vocational contexts.

Trainers

Trainer experience or qualifications required Trainers must have experience of: numeracy / maths teaching and learning in vocational contexts and teacher education/ staff development in a range of contexts, knowledge of the adult numeracy core curriculum
<http://www.excellencegateway.org.uk/node/1514> and functional skills subject criteria:
<http://www2.ofqual.gov.uk/downloads/category/68-functional-skills-subject-criteria> .

Reference material for trainers Trainer notes

Resources

Resources for reference during the session Trainer notes
Adult numeracy core curriculum
<http://www.excellencegateway.org.uk/node/1514>
Functional skills subject criteria for mathematics (downloadable from:
<http://www.ofqual.gov.uk/downloads/category/68-functional-skills-subject-criteria>).

Pre-course activity for participants Participants should bring a copy of a summative assessment for Functional Mathematics Level 1 used in their own organisations.

Useful websites See HO 7: Further useful websites

Functional skills starter kit:
<http://www.excellencegateway.org.uk/node/20280>

Adult numeracy core curriculum:
<http://www.excellencegateway.org.uk/sflcurriculum>

Before the session the trainer needs to: Print copies of the Participant pack, which contains the handouts and PowerPoint slide notes (but not the resources).

Identify and prepare materials required for activities:

TN 1: Prepare R 1: Equivalence card match. Please note when preparing the cards that the dot over the 0.6 (match to $\frac{2}{3}$) shows a recurring decimal. You may have to put it in by hand if not.

TN 3: Ideally link directly to the adult numeracy core curriculum on <http://www.excellencegateway.org.uk/sflcurriculum> and briefly demonstrate how it works and what it contains.

TN 5a: Print on card R 2: Target number template and R 3: Digit cards. Cut up R 3.

TN 5b: Prepare R 4 Sometimes, Always, Never true cards.

TN 5c: Prepare the following:

- 5m length of string
- Blank pieces of paper, about A6 size
- Supply of paper clips, dressmaking pins, or clothes pegs

TN 5d: Print on card and cut up cards for R 5: Choosing the can.

TN 6: print one copy each of the 9 questions plus example learner responses in R 6: Analysing errors. Ideally printed on A3.

TN 7: print copies of R 7 and R 8.

Session plan

Aims

To enable a conceptual understanding of number to be developed with learners in vocational contexts

Outcomes

By the end of the session participants will have:

- identified some techniques and strategies used in calculating and how these can build towards a conceptual understanding of number;
- experienced a range of maths learning activities and have adapted one (or more) to suit relevant vocational contexts;
- reviewed techniques that expose common learner errors and misconceptions in number and identified how to use these to progress learning and understanding; and
- identified challenges to learners presented by functional mathematics accreditation and explored how these may be overcome.

Suggested timings are for guidance purposes only. Trainers should adapt content to meet the needs and experience of the participants.

TN – trainer notes HO – handout R – resources PPT – PowerPoint

Time	Content	Resources		
		No.	Style	Title
15m	TN 1: Starter: Equivalence Card Match Common fractions, decimal fractions and percentages. The cards in R 1 should be printed and cut up. Distribute the cards randomly to participants. Display PPT 2 . Challenge participants to find the other three people who have an alternative representation of the number on their card. Once all four are together	R 1	Resource	Equivalence card match
		PPT 2	Slide	Starter: card match

Time	Content	Resources		
		No.	Style	Title
(Total 15m)	<p>they should sit at a table and discuss the questions displayed on the slide.</p> <p>What are the barriers to making progress in mathematics? How can these be overcome?</p> <p>Take quick feedback on the activity and discussions and alert to how these barriers will be addressed during the session.</p>			
10m	<p>TN 2: Introductions (PP3-4)</p> <p>Outline the aims and intended outcomes for the day PPT 3-4 and HO 1.</p> <p>Show PPT 5. Ask participants from each group to say what they have learned about each other during the starter activity. (Or ask participants to introduce themselves if introductions have been done first).</p> <p>Identify what vocational work and teaching experiences are represented in the group. With reference to the aims and intended outcomes, discuss what they would like to gain from the day. Note issues that need to be returned to later in the training.</p> <p>Refer to the focus of the module as being the development of a conceptual understanding of maths rather than the memorising of different methods employed in processing number. Underline how this fits with a functional approach to maths in which representing, analysing and interpreting are key processes.</p>	<p>PPT 3-5</p> <p>HO 1</p>	<p>Slides</p> <p>Handout</p>	<p>Aims and outcomes</p> <p>Aims and outcomes</p>
(Total 25m)	<p>Draw participants' attention to HO 2: Personal reflection sheet and confirm that they should complete this during each part of the session.</p>	HO 2	Handout	Personal reflections
15m	<p>TN 3: Establishing the relevance of vocational contexts in teaching functional mathematics.</p> <p>Introduce the adult numeracy core curriculum (ANCC) with PPT 6. Ideally, link to the interactive on-line curriculum: http://www.excellencegateway.org.uk/sflcu</p>	PPT 6	Slide	Adult numeracy core

Time	Content	Resources		
		No.	Style	Title
(Total 40m)	<p>riculum and demonstrate its content, referring to key documents within it. If participants have already attended Module 12a: Handling data then this part can be omitted or just provide a quick revision.</p> <p>Give a brief overview of the ANCC with the slide as a backdrop. Refer to the development of the curriculum as part of the Skills for Life strategy in 2000. Point out that the functional skills subject criteria draw heavily on the ANCC. The diagram gives an outline of the coverage and range of numeracy / maths.</p> <p>Ask participants to contribute vocational examples for each of the three strands (i.e. number, measures, shape and space and handling data).</p> <p>Leave PPT 6 displayed and draw participants' attention to HO 3 - What's involved in Number.</p> <p>Ask participants to work singly or in vocationally-related pairs to identify aspects of number relevant to their learners and their programmes of study. Encourage them to make connections across the strands of the curriculum to show how the number skills are utilised in the wider sense, e.g. multiplication used to calculate area.</p> <p>If time allows, encourage participants to extend the discussion across vocational settings to gain understanding and insight into other contexts. Compare this with the 'familiarity' of functional skills.</p>	HO 3	Handout	What's involved in number
30m	<p>TN 4. Identifying existing skills in calculating</p> <p>Introduce this activity by recognising that participants will have been taught different calculation methods and developed their own strategies over the years. Explain that the point of the activity is to explore and share these different approaches.</p> <p>Draw participants' attention to HO 4 and</p>	HO 4	Handout	Calculations

Time	Content	Resources		
		No.	Style	Title
(Total 1hr 10m)	<p>give participants 5 minutes to work through the questions. Support where appropriate.</p> <p>Go through the questions one at a time asking for volunteers to offer method of attack. Then ask for alternative approaches. If alternatives are not offered then supply them until all / most common approaches have been covered. Carefully question to explore the understanding of the methods used, particularly in subtraction and multiplication.</p> <p>Show PPT 7 and explore the slide, asking participants to give examples to illustrate the relationships.</p>	PPT 7	Slide	Connecting the four rules
20m	<p>TN 5. Developing conceptual understanding</p> <p>Explain that this section is about developing participant’s own conceptual understanding and modelling ways they can progress the understanding of their learners.</p> <p>For each activity (5a, 5b, 5c and 5d) ask participants how they might use and adapt the material with their learners.</p> <p>Activity 5a. Place value and a feel for number</p> <p>Hand out R 2: Target number templates and explain the first round. There are 10 cards with the single digits 0 – 9 (these are made from R 3). A card will be selected at random and they must write that digit in one of the four boxes on the first row. This will be repeated three times. The effect will be to give them 2 two-digit numbers which they then add together. The challenge is to get their answer as close to 100 as they can.</p> <p>Complete first round, ensuring that participants write the digits down as they are called out. Check answers and discuss strategies.</p> <p>Repeat. Ask how strategies had changed.</p>	R 2	Resource	Target number template
		R 3	Resource	Digit cards

Time	Content	Resources		
		No.	Style	Title
(Total 1hr 30m)	<p>Continue as time allows varying the target answer. (See TN 5: Trainer notes below)</p> <p>Allow 5-minute on-table discussion to consider the mathematical concepts that are highlighted by this activity and how it can help learners develop a 'feel' for number.</p> <p>Take feedback and record points on flip chart, eliciting:</p> <ul style="list-style-type: none"> • Place value • Probability • Estimation • Approximation • Problem solving and reasoning • Multiples and factors • Addition and multiplication are commutative but it is worth pointing out that whilst $24 + 36 = 26 + 34$ <p>it should be noted that: $24 \times 36 \neq 26 \times 34$</p> <p>Be ready to follow up any of the points which need expanding.</p>			
15m (Total 1 hr 45m)	<p>TN 5. Developing conceptual understanding</p> <p>Activity 5b. Sometimes, Always, Never True</p> <p>Paired activity. Allocate one or more cards made from R 4 to each pair and ask them to consider whether the statement is sometimes always or never true.</p> <p>Encourage participants to explore their response by experimenting with a variety of numbers or values.</p> <p>Small group activity. Give each group the whole set of cards and ask them to group them under Sometimes, Always, Never headings.</p> <p>In both cases review the responses and encourage participants to challenge any they do not agree with. Highlight any</p>	R 4	Resource	Sometimes, Always, Never cards

Time	Content	Resources		
		No.	Style	Title
	<p>misconceptions, ask participants to offer additional misconceptions they have observed with their learners.</p> <p>As with all the activities in this section encourage participants to comment on the value of such an activity in concept development and how they may use the activity in their own vocational setting.</p>			
15m	<p>TN 5. Developing conceptual understanding</p> <p>Activity 5c. Ordering and equivalence Fix a piece of string between two points where it can be seen by all participants. At one end of the string, pin up a piece of paper showing a zero, at the other a piece of paper showing 1. Ensure there is a pile of small pieces of paper and some paper clips on each table.</p> <p>Explain that the purpose of the exercise is to develop a concept of positive numbers less than 1. Ask each participant to write a number between 0 and 1 on a piece of paper. Invite them to pin their number in the appropriate position on the string.</p> <p>When all numbers have been placed, ask them to review the numbers and decide whether any of them should be moved; encourage them to give reasons for their decision.</p> <p>Challenge participants to add more numbers to fill gaps and to match those already there with alternative representations. Ensure that both decimals and fractions are included.</p> <p>Encourage reflection by posing the following questions:-</p> <ul style="list-style-type: none"> • What is the value of this type of concrete activity for concept development? Why? • How might the activity be adapted for other mathematical concepts? • How might you adapt it for your vocational area? 		<p>5m length of string</p> <p>Blank pieces of paper, about A6 size</p> <p>Supply of paper clips, dressmaking pins, or clothes pegs</p>	
(Total 2hr)				
15m	TN 5. Developing conceptual			

Time	Content	Resources		
		No.	Style	Title
(Total 2 hr 15m)	<p>understanding</p> <p>Activity 5d. Interpreting answers Use this activity to explore the widely-held view that maths is about right and wrong answers. Functional maths emphasises the importance of interpreting answers to provide solutions to problems. This may require a radical change in attitude and approach for both teachers and learners alike.</p> <p>Introduce the activity by explaining that someone is feeling thirsty and wants to buy a can of drink - make it clear that there is only one person wanting a can.</p> <p>Distribute copies of cards made from R 5 - Choosing the can. Ask participants to number the boxes in the order as to which is the most appropriate choice of can in the circumstances. Give time to complete any necessary calculations and make decisions.</p> <p>After about 5 minutes, ask participants to share their solution with others on the table. Pose the following questions:</p> <ul style="list-style-type: none"> • How did they work out equivalent costs? • Did everyone put them in the same order? • What arguments did they employ for their selection? • How does this activity differ from 'closed' calculations with set answers? • How could they adapt this activity for their own context? 	R 5	Resource	Choosing the can
30m	<p>TN 6. Exploration of common errors</p> <p>Show PPT 8 - and analyse this common error as a whole group. Identify the possible misconceptions and approach adopted by the learner.</p> <p>Explore reasons for the errors and misconceptions developing. Show PPT 9 which summarises reasons for errors. Elicit strategies for progressing learning</p>	PPT 8 PPT 9	Slide Slide	Common errors and misconceptions Misconceptions and errors

Time	Content	Resources		
		No.	Style	Title
(Total 2h 45m)	and understanding. Display PPT 10 which gives instructions for the 'Errors Carousel' – resource made from R 7: Analysing errors .	PPT 10	Slide	Misconceptions and errors instructions
	Give out one sample error sheet, prepared from R 7 (9 questions with sample learner answers) per group of 2–3. Each group makes notes, then leaves their results out on their table and goes round to look at other people's, adding further comments if needed. Allow about 15minutes for participants to circulate. Take feedback: <ul style="list-style-type: none"> • Have they seen these types of errors with their learners? • How has what they've learned today changed their approach to supporting the learner? • Do they feel more confident of supporting their learners? Why? 	R 6	Resource	Analysing errors - one per A3 sheet
	Draw participants' attention to HO 5 - Sample errors and suggestions	HO 5	Handout	Sample errors
	Follow up questions: <ul style="list-style-type: none"> • What can be gained from exploring errors and misconceptions? • How could this be done with a group of learners? 			
20m	TN 7. Proportional reasoning Explain that this activity is about proportional reasoning and ask participants to give examples of direct and inverse proportion (ensure participants are clear about terminology). Distribute R 7 - Proportional reasoning and ask participants to work in pairs to answer the 'paint' questions using a calculator. After a short while hold a discussion on the different approaches for each question. Ask the question:	R 7	Resource	Proportional reasoning

Time	Content	Resources		
		No.	Style	Title
(Total 3h 5m)	<p>Is there just one method that can be used for all the paint questions?</p> <p>Ask participants to complete the 'footballer' question in a similar way. Ask for solutions.</p> <p>What single method can be used to answer all the 'footballer' questions? Ask participants to suggest where the difficulty lies with this type of problem. Underline that that the operation required depends on the structure of the situation not the numbers involved.</p> <p>Review the examples and demonstrate that the operation can be identified using simple numbers and then the same operation can be applied with more complex numbers.</p> <p>Distribute R 8: Supporting progress. Ask participants to work through the sheet, following the instructions and considering the questions posed.</p> <p>Ask participants to contribute real examples of the concept of proportional reasoning from their own experience.</p>	R 8	Resource	Supporting progress
25m	<p>TN 8. Summary and next steps</p> <p>Refer back to the session outcomes (PPT 4) and summarise what has been covered in the session.</p> <p>Ask participants to reflect on what they have learned and what they feel they can apply to their own practice. PPT 11 may be useful Complete HO 2.</p> <p>Ask them to share their reflections within their table and come up with three key suggestions as to how they can improve their support of learners.</p> <p>Take feedback and draw participants' attention to HO 6.</p> <p>Draw participants' attention to HO 7: Useful websites.</p>	<p>PPT 4</p> <p>PPT 11</p> <p>HO 2</p> <p>HO 6</p> <p>HO 7</p>	<p>Slide</p> <p>Slide</p> <p>Handout</p> <p>Handout</p> <p>Handout</p>	<p>Learning outcomes</p> <p>Reflection</p> <p>Personal reflection sheet</p> <p>Tips for support</p> <p>Useful</p>
(Total 3h 30m)				

Time	Content	Resources		
		No.	Style	Title
	<p>If relevant, refer to what will be covered in Module 12c.</p> <p>Thank participants for their contributions and ask them to complete an evaluation form, if required.</p>		Evaluation forms	websites

Trainer notes

These notes are to support trainers to facilitate the different activities in the module. They are not meant to be prescriptive, and trainers can adapt the activities as needed to suit the participants. Some activities can be omitted, and others extended, according to the group.

Suggestions for **alternatives**, or for **differentiation** strategies (according to the background and experiences of participants) are given in boxes in the notes for each activity.

PowerPoint slides, resources and handouts can be adapted or omitted as needed. Where appropriate the instructions for the activities are given on the PowerPoint slides. Trainers can decide to show the instructions on PowerPoint or to print off the 'instructions' slides and lay copies on tables instead or in addition.

All of the running times are suggestions. Each training session can be adapted to the group the trainer is working with and the timings adjusted accordingly.

TN 1

Trainer notes

Time	Content	Resources		
		No.	Style	Title
15m	<p>TN 1: Starter: Equivalence Card Match</p> <p>Common fractions, decimal fractions and percentages.</p> <p>The cards in R 1 should be printed and cut up. Distribute the cards randomly to participants.</p> <p>Display PPT 2. Challenge participants to find the other three people who have an alternative representation of the number on their card. Once all four are together they should sit at a table and discuss the questions displayed on the slide.</p> <p>What are the barriers to making progress in mathematics? How can these be overcome?</p>	R 1	Resource	Equivalence card match
(Total 15m)	<p>Take quick feedback on the activity and discussions and alert to how these barriers will be addressed during the session.</p>	PPT 2	Slide	Starter: card match

Alternative: you may wish to do the introductions, aims and objectives first, before the starter.

Adjust the number of cards depending on the number of participants. This activity can be used to form random groups. If groups of three are more appropriate then one category of the equivalence cards can be omitted.

The purpose of this activity is to encourage participants to focus on the concept of equivalent representation of numbers. Typically learners see fractions, decimals and percentages as unrelated topics that have no bearing on other aspects of maths. This starter activity uses commonly-occurring fractions to explore the concept of alternative forms of representation.

Tell participants that there are three other people in the room who have a different representation of the number they have on their card. Allow participants to sort out how they are going to find others in their group. Observe the individual approaches employed – see differentiation. Offer assistance if required and give clues if necessary.

Once the groups are established give time for discussion.

Take feedback on the activity, ask which equivalences were easier, which harder and why. Reiterate that fractions, decimals and percentages are not different topics but ways of representing the same thing. Check understanding of recurring decimals and the way this is indicated.

Take feedback on barriers. Discuss barriers that participants themselves may have and where they may have come from, e.g. compartmentalised teaching, complexity of fraction calculations in school, lack of conceptual understanding, lack of apparent relevance. Give reassurance that this module will help them to develop their own and learners' concept of number and will focus on the relevance of number in their vocational setting.

Differentiation

Some participants may find this activity challenging and will wait to 'be found'; some will be proactive and investigate cards held by others. Careful observation at this point will assist the trainer in identifying individual needs. If the make-up of the group is known beforehand more challenging equivalence cards can be prepared.

TN 2

Trainer notes

Time	Content	Resources		
		No.	Style	Title
10m	<p>TN 2: Introductions</p> <p>Outline the aims and intended outcomes for the day PPT 3-4 and HO 1.</p> <p>Show PPT 5. Ask participants from each group to say what they have learned about each other during the starter activity. (Or ask participants to introduce themselves if introductions have been done first).</p> <p>Identify what vocational work and teaching experiences are represented in the group. With reference to the aims and intended outcomes, discuss what they would like to gain from the day. Note issues that need to be returned to later in the training.</p> <p>Refer to the focus of the module as being the development of a conceptual understanding of maths rather than the memorising of different methods employed in processing number. Underline how this fits with a functional approach to maths in which representing, analysing and interpreting are key processes.</p>	PPT 3-5	Slides	Aims and outcomes
		HO 1	Handout	Aims and outcomes
(Total 25m)	<p>Draw participants' attention to HO 2: Personal reflection sheet and confirm that they should complete this during each part of the session.</p>	HO 2	Handout	Personal reflections

Purpose of the activity: to introduce the session and its purpose; to give trainers the opportunity to explore participants' level of confidence and skills.

The trainer provides an introduction making connections between the participant introductions, the equivalence starter, and the module's aims and outcomes.

Draw out the barriers which many people display towards maths. Sensitive discuss that these barriers exist amongst vocational teachers as well as learners.

Emphasise that ground rules for the module will follow normal lines in that everyone should feel safe to express their doubts and concerns which will be met with a supportive, understanding attitude by all.

Emphasise that the approach to this module, and to functional maths, is about developing a 'feel' for number in which maths can be seen as a connected body of concepts which enable us to make sense of work and life, rather than a series of watertight compartments holding strange methods, magical tricks and right and wrong answers.

Terminology

In this module the use of the word 'fraction' is used to identify a number that is not whole. It therefore includes both what are usually referred to as 'fractions' and 'decimals'. It should be noted that the full names of these two forms are '**decimal** fractions' and 'common **fractions**' – in common usage only the emboldened part is used. It should be noted that percentages are related to this approach in so far as a percentage represents a fractional part but is written as whole (or mixed) number.

TN 3

Trainer notes

Time	Content	Resources		
		No.	Style	Title
15m	<p>TN 3: Establishing the relevance of vocational contexts in teaching functional mathematics.</p> <p>Introduce the adult numeracy core curriculum (ANCC) with PPT 6. Ideally, link to the interactive on-line curriculum: http://www.excellencegateway.org.uk/sfcurriculum and demonstrate its content, referring to key documents within it. If participants have already attended Module 12a: Handling data then this part can be omitted or just provide a quick revision.</p> <p>Give a brief overview of the ANCC with the slide as a backdrop. Refer to the development of the curriculum as part of the Skills for Life strategy in 2000. Point out that the functional skills subject criteria draw heavily on the ANCC. The diagram gives an outline of the coverage and range of numeracy / maths.</p> <p>Ask participants to contribute vocational examples for each of the three strands (i.e. number, measures, shape and space and handling data).</p> <p>Leave PPT 6 displayed and draw participants' attention to HO 3 - What's involved in Number.</p> <p>Ask participants to work singly or in vocationally-related pairs to identify aspects of number relevant to their learners and their programmes of study. Encourage them to make connections across the strands of the curriculum to show how the number skills are utilised in the wider sense, e.g. multiplication used to calculate area.</p> <p>If time allows, encourage participants to extend the discussion across vocational settings to gain understanding and insight</p>	PPT 6	Slide	Adult numeracy core curriculum overview
		HO 3	Handout	What's involved in number
<i>(Total 40m)</i>				

Time	Content	Resources		
		No.	Style	Title
	into other contexts. Compare this with the 'familiarity' of functional skills.			

The purpose of this activity is for participants to identify naturally-occurring maths in their own vocational area.

The design of this module has avoided using invented vocational scenarios.

There are four reasons for this:-

- 1) There is a focus in this module on number concepts.
- 2) Invented scenarios can create unnecessary confusion.
- 3) Examples, even when taken from real contexts, can only focus on a few contexts which may not be at all relevant to the participants.
- 4) It is part of the development process for participants to consider where and how number concepts and skills occur in their own vocational setting.

This short activity is therefore to give the opportunity to the participants to provide their own vocational contexts. Notes can be made by participants on **HO 3**.

It is the relevance of maths to the vocational context that needs to be described and so examples of any form of maths that are 'special' to the vocational context will be important to note. Discuss the fact that summative assessments in functional skills are not context-specific so an ability to transfer skills from one context to another is important and is the 'familiarity' level indicator of functional skills.

TN 4

Trainer notes

Time	Content	Resources		
		No.	Style	Title
30m (Total 1hr 10m)	TN 4. Identifying existing skills in calculating Introduce this activity by recognising that participants will have been taught different calculation methods and developed their own strategies over the years. Explain that the point of the activity is to explore and share these different approaches. Draw participants' attention to HO 4 and give participants 5 minutes to work through the questions. Support where appropriate. Go through the questions one at a time asking for volunteers to offer method of attack. Then ask for alternative approaches. If alternatives are not offered then supply them until all / most common approaches have been covered. Carefully question to explore the understanding of the methods used, particularly in subtraction and multiplication. Show PPT 7 and explore the slide, asking participants to give examples to illustrate the relationships.	HO 4 PPT 7	Handout Slide	Calculations Connecting the four rules

Purpose of this activity: to help participants become aware of the wide variety of methods used when undertaking calculations. The activity can be used to show how some will apply 'rules' with little understanding of the concept behind these rules whilst others will be demonstrating a 'feel' for number and the ability to recognise connections and patterns.

Examples of each of these two approaches may be subtraction using the 'rule' of equal addition but with no understanding of why it works; and doubling then doubling again as a way of multiplying by 4.

Encourage participants to work through the questions in **HO 4** as quickly as they can, thinking about their strategy rather than focusing on the correct answer.

Differentiation

Be ready to offer reassurance and encouragement. Suggest participants complete the easier questions first as time is limited. Reinforce that the purpose of the activity is to support the development of conceptual understanding not to catch people out. Have calculators available and suggest participants use them if required.

Those who complete the calculations quickly can be asked to find alternative methods for each one.

When exploring the different approaches used, be prepared for some to be completely unfamiliar to the participants, e.g. chunking for division, lattice approach to multiplication.

Encourage discussion of the place for mental and pen and paper arithmetic in the light of the expectation that calculators will be used in functional skills summative assessments. How do these mental and pen and paper approaches build a conceptual understanding of, and feel for, number? Elicit the importance of the need to interpret answers, check calculations, approximate and estimate.

Encourage participants to familiarise themselves with a variety of different methods of calculation so that they are in a position to support their learners from the learners' starting point rather than relying on their own preferred method. Refer participants to the following resources:

BBC Skillswise

<http://www.bbc.co.uk/skillswise/topic-group/calculation>

Move On Maths – Methods module

http://www.move-on.org.uk/downloadsFile/download868/Maths_methods_module.pdf

PPT 7 shows the relationship between the four rules. Being functional in mathematics is not merely about being able to carry out the four rules; it requires an understanding of how the four rules are related and an ability to select and apply them appropriately. This interdependence is demonstrated in the different methods employed in calculation and is an indication of appropriate checking procedures.

Additional activity

Allocate four of the calculations to each table and ask them to devise a question from their own context to fit each calculation.

TN 5

5a. Place value and a feel for number

Trainer notes

Time	Content	Resources		
		No.	Style	Title
20m	<p>TN 5. Developing conceptual understanding</p> <p>Explain that this section is about developing participant's own conceptual understanding and modelling ways they can progress the understanding of their learners.</p> <p>For each activity (5a, 5b, 5c and 5d) ask participants how they might use and adapt the material with their learners.</p> <p>Activity 5a. Place value and a feel for number</p> <p>Hand out R 2: Target number templates and explain the first round. There are 10 cards with the single digits 0 – 9 (these are made from R 3). A card will be selected at random and they must write that digit in one of the four boxes on the first row. This will be repeated three times. The effect will be to give them 2 two-digit numbers which they then add together. The challenge is to get their answer as close to 100 as they can.</p> <p>Complete first round, ensuring that participants write the digits down as they are called out. Check answers and discuss strategies.</p> <p>Repeat. Ask how strategies had changed.</p> <p>Continue as time allows varying the target answer. (See TN 5: Trainer notes below)</p> <p>Allow 5-minute on-table discussion to consider the mathematical concepts that are highlighted by this activity and how it can help learners develop a 'feel' for number.</p>	R 2	Resource	Target number template
		R 3	Resource	Digit cards

Time	Content	Resources		
		No.	Style	Title
(Total 1hr 30m)	<p>Take feedback and record points on flip chart, eliciting:</p> <ul style="list-style-type: none"> • Place value • Probability • Estimation • Approximation • Problem solving and reasoning • Multiples and factors • Addition and multiplication are commutative but it is worth pointing out that whilst $24 + 36 = 26 + 34$ <p>it should be noted that: $24 \times 36 \neq 26 \times 34$</p> <p>Be ready to follow up any of the points which need expanding.</p>			

Activity 5a: Target numbers

This activity is adapted from Maths4Life's *Thinking Through Mathematics: Strategies for teaching and learning* (NRDC, 2007)

http://sflip.excellencegateway.org.uk/pdf/DA_Managing+the+mathematics+footprint+i+n+educational+organisations.pdf

The purpose of this activity is to help participants get a feel for number, to explore the concept of place value, to use judgement in estimation and approximation, and to model an activity they could use with their own learners.

Hand out **R 2: Target number template** – one per pair / small group. The digit cards (**R 3**) should be printed off and cut up before the activity. The trainer should take care to explain that the digit cards can be used once only in any given round – they are selected at random and not replaced. It is this aspect which introduces the concept of probability when making judgements about the placing of digits in the 'sum'.

Differentiation

Be ready for some participants to say they have no idea where to put the digits. Take feedback on strategies sensitively. Comment on how many of us follow the 'rules' of calculation without thinking about or with no knowledge of the underlying concepts or anticipating the answer.

Experience of this activity with vocational tutors suggests that participants quickly learn from each other and are eager to get closer to the target answer by considering

place value, factors, estimates etc. Enthusiasm about the value of these concepts is often expressed particularly as the activity progresses through the stages suggested below.

Depending on the participants more complex targets can be set.

Possible targets

1. 100
2. 100
3. 50

After these first three rounds the operation sign has been omitted to allow participants to make the choice. Note that some may still put in an addition for round 4 even though the target is 600.

4. 600
5. Highest number
6. Smallest (positive)
7. Negative

Put emphasis on the discussion part of this activity. Ask participants to consider the impact of their experience in terms of their support for their own learners. Many learners tackle calculations by applying one operation or another. They need to be encouraged to consider the problem and what the answer may look like. Making estimates, rounding, drawing on their knowledge of what numbers 'do' and making connections with what they know to be sensible, all help a successful problem solving approach.

TN 5

5b. Sometimes, Always, Never True

Trainer notes

Time	Content	Resources		
		No.	Style	Title
15m (Total 1 hr 45m)	<p>TN 5. Developing conceptual understanding</p> <p>Activity 5b. Sometimes, Always, Never True</p> <p>Paired activity. Allocate one or more cards made from R 4 to each pair and ask them to consider whether the statement is sometimes always or never true.</p> <p>Encourage participants to explore their response by experimenting with a variety of numbers or values.</p> <p>Small group activity. Give each group the whole set of cards and ask them to group them under Sometimes, Always, Never headings.</p> <p>In both cases review the responses and encourage participants to challenge any they do not agree with. Highlight any misconceptions, ask participants to offer additional misconceptions they have observed with their learners.</p> <p>As with all the activities in this section encourage participants to comment on the value of such an activity in concept development and how they may use the activity in their own vocational setting.</p>	R 4	Resource	Sometimes, Always, Never cards

Activity 5b Sometimes, Always, Never

This activity is adapted from Maths4Life's *Thinking Through Mathematics: Strategies for teaching and learning* (NRDC, 2007)

http://sflip.excellencegateway.org.uk/pdf/DA_Managing+the+mathematics+footprint+in+educational+organisations.pdf

The purpose of this activity is to encourage participants to explore mathematical concepts, to bring to light any misconceptions about number operations and to address them; and to model approaches they may use to support their learners.

Differentiation

This activity can be approached in different ways depending on the make-up of the group. By now the trainer will have a reasonable knowledge of the participants, their confidence and their mathematical skills. The statements can be distributed on the basis of this knowledge with the more challenging statements being given to the more confident participants. Alternatively, more confident participants can be paired with those who are less confident.

Review and print off the Sometimes, Always, Never cards. Select those most appropriate for the group. It will be necessary to have one set of cards per group or a couple of cards per pair depending on how the activity is undertaken.

Take care to observe and listen to what is happening in the groups. Typically, more confident learners will make an instant decision and carry the rest of the group with them.

Be ready to ask:

- Can you give an example of when it is true?
- Can anyone think of an example of when it's not true?
- If it's sometimes true, can you give examples of when it is and when it isn't?
- Does it work for fractions?
- Does it work for negative numbers?

The cards on area and perimeter and birthdays have been included to show that this approach can be adopted in other situations. Discuss the value of the Sometimes, Always, Never activity and ask participants to consider where they could use it in their own vocational area.

TN 5

5c. Ordering and equivalence

Trainer notes

Time	Content	Resources		
		No.	Style	Title
15m	<p>TN 5. Developing conceptual understanding</p> <p>Activity 5c. Ordering and equivalence Fix a piece of string between two points where it can be seen by all participants. At one end of the string, pin up a piece of paper showing a zero, at the other a piece of paper showing 1. Ensure there is a pile of small pieces of paper and some paper clips on each table.</p> <p>Explain that the purpose of the exercise is to develop a concept of positive numbers less than 1. Ask each participant to write a number between 0 and 1 on a piece of paper. Invite them to pin their number in the appropriate position on the string.</p> <p>When all numbers have been placed, ask them to review the numbers and decide whether any of them should be moved; encourage them to give reasons for their decision.</p> <p>Challenge participants to add more numbers to fill gaps and to match those already there with alternative representations. Ensure that both decimals and fractions are included.</p> <p>Encourage reflection by posing the following questions:-</p> <ul style="list-style-type: none">• What is the value of this type of concrete activity for concept development.? Why?• How might the activity be adapted for other mathematical concepts?• How might you adapt it for your vocational area?			
<i>(Total 2hr)</i>				

Activity 5c: Ordering and equivalence

The purpose of this activity is to enable participants to develop a deeper understanding of the ordering and forms of representation of fractional numbers. It is another activity which helps to develop a 'feel' for number. It is intended to encourage participants to be imaginative about how they support the development of functional maths and the understanding of number-related concepts with their learners.

Resources required

- A piece of string about 5 metres in length
- A pile of blank pieces of paper, about A6 size is good
- A supply of paper clips or dressmaking pins (or clothes pegs)

Refer back to the starter activity when introducing this activity. Discuss the nature of fractions as numbers in their own right. Explore decimal notation as representations of tenths and hundredths. Use calculators to input fractions as one number divided by another and comment on the resulting decimal. Once participants have pinned up their fraction numbers it is likely that repositioning of the numbers will be required to get some semblance of accuracy in terms of the scale being used. Encourage participants to view this as a problem solving task in itself. How would they manage this with a group of learners?

Focus on simple fractions that participants are likely to encounter in everyday life and work. Ask for examples of where participants encounter fractions and decimals in their vocational context. This is an opportunity to show their relevance to, for example: measuring time, weight, length; calculating - money, time; interpreting data in charts and graphs.

Differentiation

It may be necessary to encourage participants to extend their decimal fractions beyond one decimal place or common fraction equivalents.

Challenge more confident participants to come up with very small decimals and consider their position on the scale.

Continue funneling down to explore hundredths.

Other mathematical concepts that can be explored in a similar way include:

- Order and compare whole numbers
- Order and compare common measures
- Rounding
- Probability
- Averages - mean, median and mode and range
- Scale.

Possible vocational applications:

- pH scale
- Screw dimensions
- Heights of horses
- Any of the mathematical concepts mentioned above applied in a practical setting.

TN 5

5d. Interpreting answers

Trainer notes

Time	Content	Resources		
		No.	Style	Title
15m	<p>TN 5. Developing conceptual understanding</p> <p>Activity 5d. Interpreting answers Use this activity to explore the widely-held view that maths is about right and wrong answers. Functional maths emphasises the importance of interpreting answers to provide solutions to problems. This may require a radical change in attitude and approach for both teachers and learners alike.</p> <p>Introduce the activity by explaining that someone is feeling thirsty and wants to buy a can of drink - make it clear that there is only one person wanting a can.</p> <p>Distribute copies of cards made from R 5 - Choosing the can. Ask participants to number the boxes in the order as to which is the most appropriate choice of can in the circumstances. Give time to complete any necessary calculations and make decisions.</p> <p>After about 5 minutes, ask participants to share their solution with others on the table. Pose the following questions:</p> <ul style="list-style-type: none"> • How did they work out equivalent costs? • Did everyone put them in the same order? • What arguments did they employ for their selection? • How does this activity differ from 'closed' calculations with set answers? • How could they adapt this activity for their own context? 	R 5	Resource	Choosing the can
(Total 2 hr 15m)				

Activity 5d: Interpreting answers

This activity is taken from *Teaching and Learning Functional Mathematics: Resources to support the pilot of functional skills*. The whole document can be downloaded from:

<http://archive.excellencegateway.org.uk/pdf/TandLMathematicsHT281107.pdf>

Purpose: is to encourage participants to engage with functional skill approaches requiring a focus on how problems are tackled, rather than a focus on whether the answer is correct or not.

In this activity participants will be representing, analysing and interpreting as required by the process skills in the functional skills criteria. They have to decide what mathematical strategies to adopt, carry them out and make a reasoned decision about the outcomes.

Encourage participants to share their approach to the problem and be ready to justify their solutions. How did they deal with the two different-sized cans? Explore the strategies used and draw out the concept of equivalent cost per same-sized can. What other factors did they take into consideration when reaching their decisions? Make the point that functional skills summative assessments include an aspect of exactly this sort of reasoning. Often there is no 'right' answer but learners will be expected to be able to make reasoned arguments for the processes they used and their final solutions.

As before, ask for vocational examples that mirror this activity.

Encourage participants to further develop their knowledge and understanding of number concepts through the following very accessible resources which are full of tips and ideas as well as emphasising relevance to work and life:

NRDC Publication – **Fractions** by Rachel McLeod and Barbara Newmarch
Downloadable from: http://www.nrdc.org.uk/publications_details.asp?ID=69

NRDC Publication – **Number** by Barbara Newmarch and Tracy Part
Downloadable from: http://www.nrdc.org.uk/publications_details.asp?ID=105

Both these resources were developed as part of the Maths4Life project. Other booklets in the series include:

- Measurement
- Time and Money

TN 6

Trainer notes

Time	Content	Resources		
		No.	Style	Title
30m	<p>TN 6. Exploration of common errors</p> <p>Show PPT 8 - and analyse this common error as a whole group. Identify the possible misconceptions and approach adopted by the learner.</p> <p>Explore reasons for the errors and misconceptions developing. Show PPT 9 which summarises reasons for errors. Elicit strategies for progressing learning and understanding.</p> <p>Display PPT 10 which gives instructions for the 'Errors Carousel' – resource made from R 7: Analysing errors.</p> <p>Give out one sample error sheet, prepared from R 7 (9 questions with sample learner answers) per group of 2–3. Each group makes notes, then leaves their results out on their table and goes round to look at other people's, adding further comments if needed.</p> <p>Allow about 15minutes for participants to circulate.</p> <p>Take feedback:</p> <ul style="list-style-type: none"> • Have they seen these types of errors with their learners? • How has what they've learned today changed their approach to supporting the learner? • Do they feel more confident of supporting their learners? Why? <p>Draw participants' attention to HO 5 - Sample errors and suggestions.</p>	PPT 8	Slide	Common errors and misconceptions
		PPT 9	Slide	Misconceptions and errors
		PPT 10	Slide	Misconceptions and errors instructions
		R 6	Resource	Analysing errors - one per A3 sheet
		HO 5	Handout	Sample errors
(Total 2h 45m)	<p>Follow up questions:</p> <ul style="list-style-type: none"> • What can be gained from exploring errors and misconceptions? • How could this be done with a group of 			

Time	Content	Resources		
		No.	Style	Title
	learners?			

The purpose of this activity is to show how exploring errors and misconceptions can lead to a greater understanding of underlying mathematical concepts. Participants are likely to hold some of these misconceptions themselves and these may have already emerged during the course of the session. Point out that other activities today will have exposed misconceptions, e.g. sometimes, always, never. Analysing actual errors is merely one way - careful questioning is always available.

Look at **PPT 8** and ask participants for more examples of misconceptions and why they think they might have developed. Point out that some 'rules of thumb' do work but are limited by the context, e.g. to multiply by 10 you add a nought works but only for whole numbers; if you multiply two numbers together the answer is bigger than both the original numbers is true but only if both numbers are greater than 1. **PPT 9** gives some examples of common misconceptions.

PPT 10 gives instructions for the activity which uses **R 6: Analysing errors** – examples of errors made by learners. Participants are required to consider what is going on in each of these errors and to consider how to tackle them. **HO 5** gives detailed ideas of the error and ideas for support.

If time allows, discuss what effect the use of a calculator in functional maths will have on the sort of errors and misconceptions that will occur.

The active approaches to learning exemplified in these CPD modules are designed to encourage discussion, reasoning, evaluation and analysis. There is a growing body of evidence (see below) to show this brings about the 'cognitive conflict' that can result in deeper, more profound learning which enables real progress to be made.

Trainers wishing to explore this in more detail can refer to:

Swan, M. (2005) *Improving Learning in Mathematics: Challenges and Strategies*.

London: DfES. Available to download from

https://www.ncetm.org.uk/files/224/improving_learning_in_mathematicsi.pdf

Differentiation

To shorten the activity (and to make it less demanding on participants), give out only three or four sample errors, with different groups working on different ones, but some groups working on the same ones. This will cut down on feedback time but also allow for comparison of results and expose participants to a range of sample errors and types of question.

Encourage participants to think how they could make use of this type of activity with their own learners, for example:

- Learners adopt the role of exam marker – identifying errors and suggesting alternative approaches.
- Give learners examples of incorrect reasoning – their task is to correct the error and offer advice to the person who made it.
- Give learners examples of different methods and approaches to solving problems and get them to make comments in terms of efficiency of methods used and the reasoning behind them.

TN 7

Trainer notes

Time	Content	Resources		
		No.	Style	Title
20m	<p>TN 7. Proportional reasoning</p> <p>Explain that this activity is about proportional reasoning and ask participants to give examples of direct and inverse proportion (ensure participants are clear about terminology).</p> <p>Distribute R 7 - Proportional reasoning and ask participants to work in pairs to answer the 'paint' questions using a calculator.</p> <p>After a short while hold a discussion on the different approaches for each question. Ask the question: Is there just one method that can be used for all the paint questions?</p> <p>Ask participants to complete the 'footballer' question in a similar way. Ask for solutions.</p> <p>What single method can be used to answer all the 'footballer' questions? Ask participants to suggest where the difficulty lies with this type of problem. Underline that that the operation required depends on the structure of the situation not the numbers involved.</p> <p>Review the examples and demonstrate that the operation can be identified using simple numbers and then the same operation can be applied with more complex numbers.</p>	R 7	Resource	Proportional reasoning
(Total 3h 5m)	<p>Distribute R 8: Supporting progress. Ask participants to work through the sheet, following the instructions and considering the questions posed.</p> <p>Ask participants to contribute real examples of the concept of proportional reasoning from their own experience.</p>	R 8	Resource	Supporting progress

This activity is taken from Maths4Life's *Thinking Through Mathematics: Strategies for teaching and learning* (NRDC, 2007)

http://sflip.excellencegateway.org.uk/pdf/DA_Managing+the+mathematics+footprint+in+educational+organisations.pdf

Purpose of this activity: Proportional reasoning and the ability to deal with ratio are concepts common to all vocational contexts and ones which cause difficulty for tutors and learners alike. The purpose of this activity is to work through examples which require proportional reasoning and to show that the operation required lies in the structure of the problem, not in the numbers the problem contains.

Secondarily this is an opportunity for participants to see that enabling learners to work through a process with simple numbers will help them identify which operation to use when the numbers are more complex.

Start by asking participants what they understand by 'proportional reasoning' elicit direct and inverse proportion and ask for examples.

Give time for participants to work on the question in **R 7** and circulate, listening to the discussion and the approaches used. Commonly, different approaches will be used for each part of the question.

After a short while get the group together to discuss what they did. Ensure all approaches are covered by asking if anyone completed that part of the question in a different way. Once all alternatives have come to light pose the question:

- Is there just one method that can be used for all the paint questions?

Repeat the above for the 'footballer' question.

- What single method can be used to answer all the 'footballer' questions?

Ask participants to suggest where the difficulty lies with this type of problem. It is likely that it will lie in the complexity of the numbers, or the ease of 'seeing' the answer.

Underline that the operation required depends on the structure of the situation not the numbers involved.

Distribute **R 8: Supporting progress**. Ask participants to work through the sheet, following the instructions and considering the questions posed. Review the questions noting that Questions 2, 4 and 6 are incorrect. Ask for comments on the fact that the correct operation is selected in Q2 and Q4 though the numbers have been reversed – why might this be? It is likely to be that the learner thought a smaller number could not be divided by a larger one. Make connections with earlier work on concepts.

Review the examples and demonstrate that the operation can be identified using simple numbers and then the same operation can be applied with more complex numbers.

TN 8

Trainer notes

Time	Content	Resources		
		No.	Style	Title
25m	<p>TN 8. Summary and next steps</p> <p>Refer back to the session outcomes (PPT 4) and summarise what has been covered in the session.</p> <p>Ask participants to reflect on what they have learned and what they feel they can apply to their own practice. PPT 11 may be useful. Complete HO 2.</p> <p>Ask them to share their reflections within their table and come up with three key suggestions as to how they can improve their support of learners.</p> <p>Take feedback and draw participants' attention to HO 6.</p> <p>Draw participants' attention to HO 7: Useful websites.</p> <p>If relevant, refer to what will be covered in Module 12c.</p> <p>Thank participants for their contributions and ask them to complete an evaluation form, if required.</p>	PPT 4	Slide	Learning outcomes
(Total 3h 30m)		PPT 11	Slide	Reflection
		HO 2	Handout	Personal reflection sheet
		HO 6	Handout	Tips for support
		HO 7	Handout	Useful websites
			Evaluation forms	

Refer back to the session outcomes (**PPT 4**) and summarise what has been covered in the session. Ask participants to reflect on what they have learned and what they feel they can apply to their own practice. You should give them some time to complete their personal reflection sheet – **HO 2**.

After a few minutes ask them to share their reflections within their table and come up with 3 key suggestions as to how they can improve their support of learners.

Assess the time required for this discussion – it can prove to be a very valuable part of the consolidation process and should not be hurried.

Take one key point from each table at a time until no new points are made. Distribute **HO 6: Tips for support**. Encourage participants to check their own ideas against the HO.

If relevant, refer to what will be covered in Module 12c.

Refer participants to **HO 7: Useful websites** and remind them that resources from the session will be a mailed out to them.

Thank participants for their contributions and ask them to complete an evaluation form – this is not included in the trainer pack as organisations tend to use their own standard form.

Resources and handouts

Resources

- R 1: Equivalence card match
- R 2: Target number template
- R 3: Digit cards
- R 4: Sometimes, Always, Never true
- R 5: Choosing the can
- R 6: Analysing errors
- R 7: Proportional reasoning
- R 8: Supporting progress

Handouts

- HO 1: Aims and outcomes
- HO 2: Personal reflection sheet
- HO 3: What's involved in number
- HO 4: Calculations
- HO 5: Sample errors with notes
- HO 6: Tips for developing concepts and helping learners progress
- HO 7: Useful websites

R 1: Equivalence card match

$\frac{1}{5}$	0.2	One fifth	20%
$\frac{2}{3}$	0.6	Two thirds	$66\frac{2}{3}\%$
$\frac{6}{10}$	0.6	Six tenths	60%
$\frac{1}{100}$	0.01	One hundredth	1%
$\frac{3}{4}$	0.75	Three quarters	75%

R 2: Target number template

$$\square \square + \square \square =$$

$$\square \square + \square \square =$$

$$\square \square + \square \square =$$

$$\square \square + \square \square =$$

$$\square \square + \square \square =$$

$$\square \square + \square \square =$$

R 3: Digit cards

1	2	3	4	5
6	7	8	9	0

R 4: Sometimes, Always, Never true

<p style="text-align: center;">Digits</p> <p>Numbers with more digits are greater in value.</p>	<p style="text-align: center;">Add a nought</p> <p>To multiply by ten, you just add nought on the right hand end of the number.</p>
<p style="text-align: center;">Pay rise</p> <p>Max gets a pay rise of 30%. Jim gets a pay rise of 25%. So Max gets the bigger pay rise.</p>	<p style="text-align: center;">Sale</p> <p>In a sale, every price was reduced by 25%. After the sale every price was increased by 25%. So prices went back to where they started.</p>
<p style="text-align: center;">Area and perimeter</p> <p>When you cut a piece off a shape you reduce its area and perimeter.</p>	<p style="text-align: center;">Birthdays</p> <p>In a group of ten learners, the probability of two learners being born on the same day of the week is one.</p>
<p style="text-align: center;">Bigger fractions</p> <p>If you add the same number to the top and bottom of a fraction, the fraction gets bigger in value.</p>	<p style="text-align: center;">Smaller fractions</p> <p>If you divide the top and bottom of a fraction by the same number, the fraction gets smaller in value.</p>
<p style="text-align: center;">Dividing</p> <p>If you divide one number by another the answer is smaller than the original number.</p>	<p style="text-align: center;">Multiplying</p> <p>If you multiply one number by another the answer is bigger than the original.</p>

R 5: Choosing the can

This activity is taken from *Teaching and Learning Functional Mathematics: Resources to support the pilot of functional skills*.

The whole document can be downloaded from:

<http://archive.excellencegateway.org.uk/pdf/TandLMathematicsHT281107.pdf>

Pack of 4 x 330 ml cans costing £1.80	Original price for a 330 ml can is 60p Current offer is 10% off
One individual 330 ml can for 50p	One 550 ml can costing 78p
One 330 ml can costing 58p with the offer 'Buy one, get one free'	One individual 550 ml can costing 55p with the offer '50p off next purchase'

R 6: Analysing errors

Note to trainer: there are 9 questions with answers, on 9 pages. Each should be printed on a separate sheet of paper, ideally A3.

This activity is adapted from an activity in 'Helping learners progress with their maths', a CPD developed by the LSIS Skills for Life Support Programme in 2010. The session is still available on the Excellence Gateway:

<http://www.excellencegateway.org.uk/page.aspx?o=295474>

Question 1

There are 64 bottles to a crate. How many bottles are there in 17 crates?

Student answer

$$\begin{array}{r} 64 \\ \times \underline{17} \\ 68 \\ \underline{64} \\ 132 \end{array}$$

What happened and why do you think it happened?

How might you enable this learner to progress?

Question 2

£12.44 + 46p

Student answer = £12.9p

What happened and why do you think it happened?

How might you enable this learner to progress?

Question 3

What is 624 divided by 6?

Student answer
$$\begin{array}{r} 14 \\ 6 \overline{)624} \end{array}$$

What happened and why do you think it happened?

How might you enable this learner to progress?

Question 4

What is 50% of £400?

Student answer.

$$\frac{50}{400} \times 100 = \frac{50}{4} = £12.50$$

What happened and why do you think it happened?

How could you enable this learner to progress?

Question 5

Find the change from £5 for an item costing £3.97

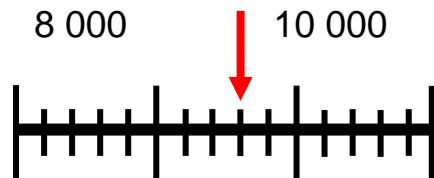
Student answer : £2.97

What happened and why do you think it happened?

How might you enable this learner to progress?

Question 6

What is the reading on the scale?



Student answer: 8 800

What happened and why do you think it happened?

How could you enable this learner to progress?

Question 7

An aircraft takes 1hr 34mins to fly 634miles. How long will it take to complete 1100miles at the same speed?

Student answer : $634 \div 1.34 = 473.13$

$1100 \div 473.13 = 2.32\text{hrs}$

What happened and why do you think it happened?

How might you enable this learner to progress?

Question 8

The label on a bottle of cleaning fluid states 'dilute 1 part cleaning fluid to 6 parts water'.

How much water must be added to 300 ml of cleaning fluid?

Student answer: 50 ml

What happened and why do you think it happened?

How might you enable this learner to progress?

Question 9

A department store is open 7 days a week, with late night opening on Thursdays. Two thirds of the employees travel to work by public transport or by walking. The rest travel by car. There are 171 employees altogether. How many travel to work by car? Show how you got your answer.

Write down two more factors that you might need to take into account when allocating parking spaces.

Student answer: 114

Some people might get dropped off

What happened and why do you think it happened?

How might you enable this learner to progress?

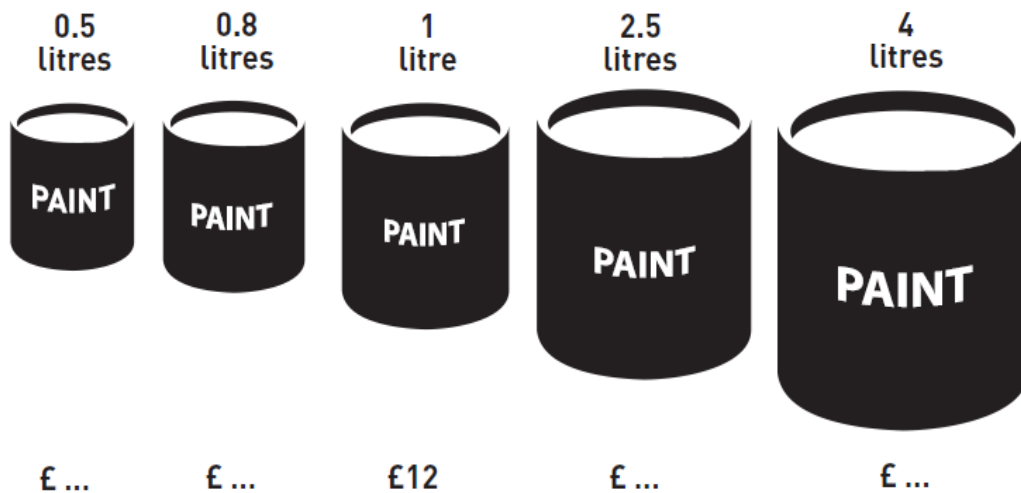
R 7: Proportional reasoning

Activity taken from the Standards Unit materials:

<http://tlp.excellencegateway.org.uk/teachingandlearning/downloads/default.aspx#/math>

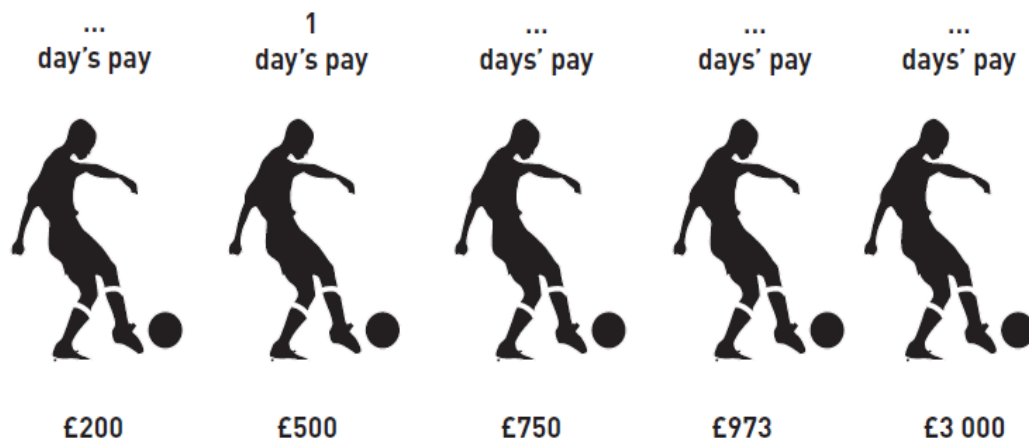
Calculate the missing prices of the paint cans.

The prices are proportional to the amount of paint in the can.



Calculate the missing days' pay for the footballer.

The pay is proportional to the number of days worked.



R 8: Supporting progress

Activity taken from the Standards Unit materials:

<http://tlp.excellencegateway.org.uk/teachingandlearning/downloads/default.aspx#/math>

A learner has been asked to write down how she would solve six problems.

Do you agree with her answers?

What mistakes has she made?

Can you correct her mistakes?

Can you see why she has made these mistakes?

1	A car travels 120 miles in 3 hours at a steady speed. How far does it go in 1 hour?	$120 \div 3$
2	A snail moving at a steady speed travels 0.8 miles in 40 hours. How far does it go in 1 hour?	$40 \div 0.8$
3	I buy some apples which cost £1.50 per kilogram. I spend £3.50. What weight do I buy?	$3.50 \div 1.50$
4	I buy some tomatoes which cost £0.90 per kilogram. I spend 30 pence. What weight do I buy?	$90 \div 30$
5	Sam's motorbike does 60 miles per litre. How far can she go on 3 litres?	60×3
6	Clive's car does 20 miles per litre. He only has 0.4 litres left in the tank. How far will he travel before he runs out of petrol?	$20 \div 0.4$

A second learner is trying to explain how to do the last problem.

If you can't decide whether a problem is multiply or divide, then try changing the numbers to easier ones.

Just change the 20 to 6 and the 0.4 to 3. Then it is easy to see that the question should be multiply.

The first learner replies:

I don't think your method works. If you change the numbers, you might change the operation.

HO 1: Aims and outcomes

Aims

To enable a conceptual understanding of number to be developed with learners in vocational contexts

Outcomes

By the end of the session participants will have:

- identified some techniques and strategies used in calculating and how these can build towards a conceptual understanding of number;
- experienced a range of maths learning activities and have adapted one (or more) to suit relevant vocational contexts;
- reviewed techniques that expose common learner errors and misconceptions in number and identified how to use these to progress learning and understanding; and
- identified challenges to learners presented by functional mathematics accreditation and explored how these may be overcome.

HO 2: Personal reflection sheet

Module 12b: Number concepts and skills

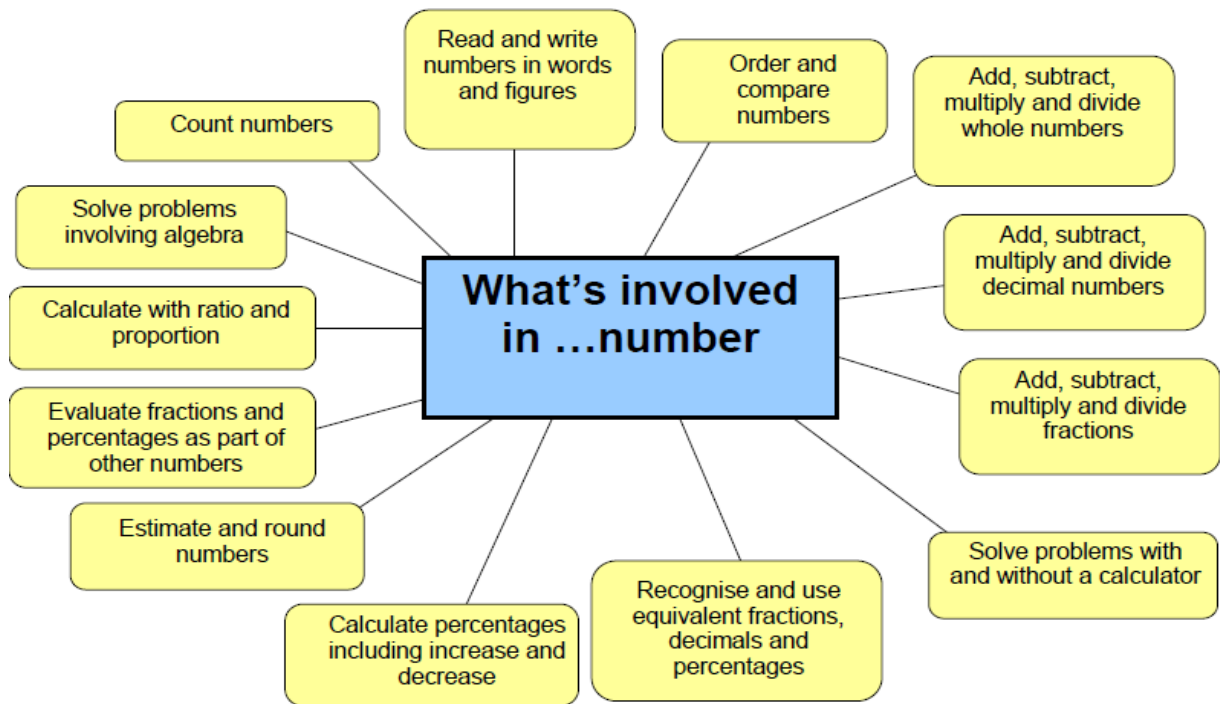
As you go through the different topics and activities during the session, make notes below regarding topics you feel confident about and those that you need to consolidate further.

Topic / activity	Reflections
Equivalence card match	
Introductions	
Vocational contexts and functional maths	
Identifying existing skills in calculating	
Place value and a feel for number	
Sometimes, Always, Never True	
Ordering and equivalence	
Interpreting answers	

Exploration of common errors	
Proportional reasoning	
Next steps	

HO 3: What's involved in number

<http://repository.excellencegateway.org.uk/fedora/objects/import-pdf:14067/datastreams/PDF/content>



Identify the situations in which these number skills are required in your vocational area.

List your examples below:

HO 4: Calculations

Calculations

1. $19 + 59$
2. $203 - 78$
3. $£10 - £5.99$
4. 60% of £120
5. 10×3.1
6. 0.75×0.2
7. $\frac{5}{8}$ of £240
8. $918 \div 17$
9. $63 \div 3$
10. 21.5×4

HO 5: Sample errors with notes

This activity is adapted from an activity in 'Helping learners progress with their maths', a CPD developed by the LSIS Skills for Life Support Programme in 2010. The session is still available on the Excellence Gateway:

<http://www.excellencegateway.org.uk/page.aspx?o=295474>

Note

The reasons and strategies suggested here are not the only possibilities – there could be other alternatives. Also, sometimes particular strategies will not work – it depends on the learner and on the context. There is no one 'cure-all' approach!

Question 1

There are 64 bottles to a crate. How many bottles are there in 17 crates?

Student answer

$$\begin{array}{r} 64 \\ \times 17 \\ \hline 68 \\ \underline{64} \\ 132 \end{array}$$

The learner has multiplied the 4 in 64 by 17. This method would have worked if they had then multiplied the 60 in 64 by 17 and added the two answers together. However, they have become muddled and multiplied 64 by 1 instead. They have also forgotten to take into account that the '1' in 17 is in fact 10, so there should have been a blank space or a zero after the answer '64' to show that it is in fact 640. This learner is having problems remembering the standard written algorithm for long multiplication. Some learners with dyslexia might find it hard to recall which order to multiply the digits in, or to remember where they are up to.

Some strategies for Question 1

Encourage the learner to find ways to check the answer, and to see if it seems 'sensible' (e.g. by visualising the crates and bottles, or by thinking that the answer must be bigger than 600 because 64 is bigger than 60 and 17 is bigger than 10).

Provide an alternative way to multiply, e.g.

$$\begin{array}{r} 64 \\ \times 17 \\ \hline 28 \quad (4 \times 7) \\ 420 \quad (60 \times 7) \\ \hline 640 \quad (64 \times 10) \\ \hline 1088 \end{array}$$

The method above breaks down the algorithm into shorter steps, but the learner still needs to remember which digit they are multiplying by and which ones are left. A

way round this is to use the grid method below (another suggestion is the lattice method):

X	60	4	
10	600	40	
7	420	28	
totals	1020	68	1088

Question 2

$$£12.44 + 46p$$

Student answer = £12.9p

The learner has used a calculator and the result is shown as 12.9. The learner has also misunderstood how to write sums of money and does not realise that the 'p' is not required yet.

Some strategies for Question 2

Any or all of these and more:

- Ask the learner to read out the answer and ask how they know if it is 9 or 90p.
- Ask the learner to add 44p and 46p using real coins and to write the answer down. Compare with the calculator results and discuss.
- Look at advertisements or catalogues to see how sums of money are recorded.
- Compare with the way that measurements are written down
- Discuss what each 'column' stands for when writing down sums of money.
- Play money 'bingo' with different sums of money written in figures on grids. The teacher reads out a list of sums of money and the learners cross them off as they hear them. The first to cross out the whole grid wins.

Question 3

What is 624 divided by 6?

Student answer
$$6 \overline{)624} \quad 14$$

Instead of thinking '6 into 2 won't go, so put a zero and carry over the 2', the learner has calculated 24 divided by 6 and written the answer at the top.

Some strategies for Question 3

Encourage checking strategies and seeing if the answer is 'sensible'. Once the learner has discovered that the answer is wrong, see if they can correct it themselves, or with peer support. Offer alternative ways of calculating division, e.g. 'chunking' (type 'division by chunking' into an internet search engine for some explanations of this, including a video on YouTube), which uses repeated subtraction.

In fact the standard written algorithm uses a similar method to chunking.

3	7	7	Step 4: This adds up to 377
		7	Step 3: 3 into 21 goes 7 times
	7	0	Step 2: 3 into 230 goes 70 times, remainder 20 (which is added o the 1)
3	0	0	Step 1: 3 into 1100 goes 300 times, remainder 200 (which is added to the 31)

$$3 \overline{) 112321}$$

This is not to suggest you would explain this to every learner, but it is helpful to understand it as a teacher.

Question 4

What is 50% of £400?

Student answer:

$$\frac{50}{400} \times 100 = \frac{50}{4} = \text{£}12.50$$

The learner has mis-remembered the algorithm for finding percentage quantities and has put the 100 on the top instead of the bottom. They are relying on rote learning of procedures rather than understanding what is really happening.

Some strategies for Question 4

This is about enabling the learner to discover the connections between fractions and percentages, and also connecting their intuitive understanding with formal procedures. Ways to proceed depend on the learner's starting point, e.g. do they understand that 50% is the same as $\frac{1}{2}$ or $\frac{50}{100}$? Offer the learner some contextualised examples where 50% is used, for example in a sale, and ask them to work out the discount mentally. Then ask them what method they used. If their method works, go back to the original example and ask them to apply the same

method. Other strategies might include using a visual representation, e.g. a diagram or a set of objects (e.g. 4 lots of 100 square grids).

Question 5

Find the change from £5 for an item costing £3.97

Student answer : £2.97

The learner has calculated this using the standard written algorithm, but instead of $0-7$, has calculated $7-0$ and got 7, and similarly for the rest of the calculation.

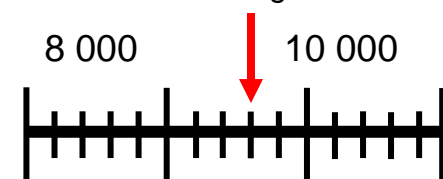
Using the standard written algorithm to work out 'change' is artificial, because people do not get out a pen and paper and work it out this way when shopping – it is usually calculated mentally or electronically.

Some strategies for Question 5

Use real money, counting on, calculators, a number line, etc. Once the learner has a secure method, you could ask them to go back to the original calculation to discuss what they did wrong.

Question 6

What is the reading on the scale?



Student answer: 8 800

The learner has looked at the 8000 and not the 10000 and has made the assumption that each division on the scale is worth 100. The learner has not understood that different scales have different scales!

Some strategies for Question 6

The learner could be asked to look at a number of 'scales', e.g. on measuring tapes, weighing scales, clocks, etc and asked to work out what each division represents on each type of scale. Once they have a method for this, they could revisit the original example.

Question 7

An aircraft takes 1hr 34 mins to fly 634 miles. How long will it take to complete 1100 miles at the same speed?

Student answer : $634 \div 1.34 = 473.13$

$1100 \div 473.13 = 2.32$ hrs

The reasoning is sound in this answer; it is the handling of time that is at fault. The learner has written the time taken as a decimal without considering that there are 60 minutes in an hour not 100. The final answer has been left as a decimal so it is not easy to see if the learner thinks this is 2 hrs 32 mins.

Some strategies for Question 7

Incorrect conversion of time to decimals is a very common error. Explore the learner's understanding of the decimal system and place value. Discuss the representation of quarter hours and half hours as decimals and how this relates to equivalences the learner may already be happy with, e.g. How would they represent $2\frac{1}{2}$ hrs as a decimal? Engage in plenty of equivalence activities with other measures so that time can be seen as a special case. (Consider the starter activity for this module and module 12c). Give practice with real life examples, e.g. bills, wage slips, time sheets where time is given in a decimal format.

Question 8

The label on a bottle of cleaning fluid states 'dilute 1 part cleaning fluid to 6 parts water'.

How much water must be added to 300 ml of cleaning fluid?

Student answer: 50 ml

The learner has divided instead of multiplied. Ratio questions have several pieces of information which some learners (e.g. dyslexic learners) find problematic to hold in their heads all at once.

Some strategies for Question 8

Try the question 'for real' using measuring jugs, water and a coloured liquid such as fruit squash! Ask the learner to write down all the important information in the question and then to think about how to set it out in a systematic way (e.g. in a table), and see if this helps.

Question 9

A department store is open 7 days a week, with late night opening on Thursdays. Two thirds of the employees travel to work by public transport or by walking. The rest travel by car. There are 171 employees altogether. How many travel to work by car? Show how you got your answer.

Write down two more factors that you might need to take into account when allocating parking spaces.

Student answer: 114
Some people might get dropped off

This is not untypical of a 'functional skills' style of question. The learner has to select the information necessary for answering the question and discard any irrelevant information (although this may be needed later). 'Working out' is asked for and a question relating to the context is included. This learner has worked out $\frac{2}{3}$ instead of $\frac{1}{3}$, and has not shown their method. However, the problem appears to be decoding the question rather than anything to do with finding fractions of quantities.

The answer to the last part is an acceptable one, and shows that they have considered the context as a whole. Other acceptable answers could be that given the long opening hours and the store opening 7 days a week, staff probably work shifts and therefore would not all be there at the same time, or that people might car-share, or that those with a disability would need car parking spaces close to the building.

Strategies for Question 9

Ask the learner to read the question again (they could highlight or write out the relevant information) and estimate the answer. A sketch of a pie chart representing the total number of employees might help.

Explain that credit is given for showing methods, and ask the learner to explain their method for this question, and how they might write it down. You might also want to give them positive feedback about the answer to the last part, and to discuss what other factors might be relevant.

HO 6: Tips for developing concepts and helping learners progress

- Talk maths.
- Make connections between mathematical ideas and concepts.
- Find a range of interesting, relevant contexts for number work.
- Ask learners to identify naturally-occurring mathematical problems and scenarios in their vocational and life contexts.
- Expose and discuss common misconceptions.
- Start where the learner is by building on what they already know.
- Use challenging, collaborative tasks to get learners talking, reasoning and justifying.
- Encourage learners to adopt a Plan / Do / Review approach to problem solving.
- Encourage reasoning rather focusing on 'right' answers.
- Make it clear that it's OK to make mistakes.
- Remind learners to estimate answers before starting a calculation.
- Make connections between what learners already know and what is new.
- Encourage learners to think forwards and backwards by giving an answer and asking them to write the question.
- Encourage a range of checking processes – reverse operations, alternative methods, use a calculator, common sense or experience.
- Encourage learners to substitute simpler numbers as a way of seeing how to approach a problem.

HO 7: Useful websites

Adult numeracy core curriculum

<http://www.excellencegateway.org.uk/node/1514>

New interactive online version. As well as the numeracy curriculum, there are sections on embedding, family learning and employability, links to resources and other curricula, ideas, suggestions and activities, personal space, contributions from other tutors and more.

BBC Skillswise

<http://www.bbc.co.uk/skillswise/maths>

Online and paper-based resources for adult numeracy learners.

Braingames

<http://www.braingames.org.uk/>

Online interactive learning materials in a games format, including English and maths.

Being Functional resources

<http://tlp.excellencegateway.org.uk/tlp/fs/fs-resources/about.php>

[A range of functional skills resources, including CPD activities.](#)

Excellence Gateway - nationally developed Skills for Life materials

<http://rwp.excellencegateway.org.uk>

Resources developed over the period 2001 to 2010 to support the national Skills for Life Strategy and other Skills for Life developments.

Embedded learning materials

<http://rwp.excellencegateway.org.uk/Embedded%20Learning/>

An extensive range of materials to support embedded learning (including numeracy) in over 25 vocational, community, employability and health settings.

Functional skills on the Excellence Gateway

<http://www.excellencegateway.org.uk/node/21154>

See this menu page to access the range of functional skills resources on the Excellence Gateway, including the new functional skills starter kit:

<http://www.excellencegateway.org.uk/node/20280>

Improving Learning in Mathematics

<http://tlp.excellencegateway.org.uk/teachingandlearning/downloads/default.aspx#/math>

Resources for improving teaching in mathematics, including a selection of downloadable materials. Aimed primarily at Level 2 and 3 learners.

Learning Mathematics in Context

<http://tlp.excellencegateway.org.uk/tlp/xcurricula/lmic/>

Ideas and resources to help you explore teaching and learning mathematics within vocational and other subject areas.

Mathematical Moments

<http://tlp.excellencegateway.org.uk/tlp/stem/stem-mm.html>

Each Mathematical Moment invites you to focus on a particular mathematical topic, offers you suggestions for activities you could carry out with your learners, prompts you to anticipate, and then reflects on learners' responses, and finally offers you some follow-up ideas. The topics are addressed at levels ranging from Entry to Level 3.

Move On

<http://www.move-on.org.uk/>

English and Maths resources for teachers, learners and providers, encompassing promotion, engagement and delivery. Check out Stop 4 of the Teacher Route.

National Centre for Excellence in the Teaching of Mathematics

<https://www.ncetm.org.uk/>

Resources and tools for teachers of maths and numeracy across all sectors (primary, secondary and FE). Check out the following pages. Note that you need to register before accessing these materials.

Numeracy Challenge <https://www.ncetm.org.uk/resources/13790>

Maths at Work <https://www.ncetm.org.uk/resources/11329>

FE Magazine <https://www.ncetm.org.uk/resources/14609>

Mathemapeda <https://www.ncetm.org.uk/mathemapeda/>

Thinking Through Maths (online CPD module)

<https://www.ncetm.org.uk/reflective-learning/ttm>

Northern College

<http://www.northern.ac.uk/content/?id=133>

Active resources for teaching functional mathematics (Entry 3 and Level 1).

Nrich

<http://nrich.maths.org/public/index.php>

Free mathematics enrichment materials (problems, articles and games) for teachers and learners. Aimed at ages 5 to 19 years, but much is suitable for adults.

OCR support materials for Functional Skills Maths:

Level 1: <http://www.ocr.org.uk/qualifications/type/fs%5F2010/maths/l1/documents/>

Level 2: <http://www.ocr.org.uk/qualifications/type/fs%5F2010/maths/l2/documents/>

Tasks to use as teaching resources or practice assignments.

Office of Fair Trading Skilled to Go

<http://www.oft.gov.uk/about-the-oft/partnership-working/partnership-working-info/consumer-education/resources/sthome>

A teacher's toolkit of games and resources for consumer education, with literacy and numeracy embedded.

Resources to support the pilot of functional skills

<http://www.excellencegateway.org.uk/page.aspx?o=201311>

Teaching and learning functional mathematics

Skills workshop

<http://www.skillsworkshop.org/>

Free downloadable Skills for Life and functional skills resources from this private website.

Subtangent

<http://www.subtangent.com>

Interactive maths games and resources that can be used on line or downloaded.

Tarsia Formulator

http://www.mmlsoft.com/index.php?option=com_content&task=view&id=4&Itemid=5

Free downloadable software to help create your own mathematical jigsaws and domino activity.