

# 6

## What chance?



“My sister, Kim, is expecting twins!

All the family are wondering whether they will be boys or girls.”

### Talk about it

What are the chances that Kim will have:

- two girls?
- two boys?
- a girl and a boy?

What do you think? Are the chances all the same?

Calculating probability can help us answer questions like this.

But probability is only theoretical. For example, the sex of a baby is influenced by factors other than chance.

This unit looks at the possible outcomes of combined events – that is, the possible result of two or more things happening, where the outcome of one is not affected by the outcome of the other. These are called **independent events**.

### These are the skills you will practise in this unit.

Which are the most useful for you? Tick the boxes.

- ☐ Identifying the range of possible outcomes of combined events and recording the information using diagrams or tables
- ☐ Using fractions to describe and compare probabilities

**Skill code**

HD2/L2.1

N2/L2.3

# Outcomes

If my sister were expecting just one baby, there would be two possible outcomes for the sex of the baby: a girl or a boy. We are going to assume that these are **equally likely** events.

## Activity 1

The table is one way of recording the outcomes of two events. It is a **sample space table**.

Along the top it shows the two possible outcomes of the sex of the first twin (G is girl, B is boy).

Down the side it shows the two possible outcomes of the sex of the second twin.

sex of second twin

|   | sex of first twin |    |
|---|-------------------|----|
|   | G                 | B  |
| G | GG                | BG |
| B | GB                | BB |

The number of possible outcomes is shown by the coloured squares inside the table.

- How many possible outcomes are there? .....
- There is only one outcome both twins will be girls, shown by the lilac square GG. The probability of two girls is ..... Write  $P(2 \text{ girls}) = \dots\dots\dots$
- $P(2 \text{ boys}) = \dots\dots\dots$
- $P(\text{a girl and a boy}) \dots\dots\dots$   
 $\frac{1}{2}$  is bigger than  $\frac{1}{4}$

so **theoretically**, my sister is more likely to have a girl and a boy, than two girls or two boys.

What is the probability that my sister will have twins of the same sex?

.....

## Activity 2

There are three primary colours: blue, yellow and red. When different colours are mixed together, new colours are produced. Kim has two boxes each containing tins of the three primary colour paints. Kim is going to select a tin from each box and mix them together. Complete the table to show the possible results.

|               |            | First colour |            |         |
|---------------|------------|--------------|------------|---------|
|               |            | Blue (B)     | Yellow (Y) | Red (R) |
| Second colour | Blue (B)   | BB           | YB         | RB      |
|               | Yellow (Y) | BY           |            |         |
|               | Red (R)    |              |            |         |

Use the table to answer these questions.

- Using the three primary colours how many ways can two colours be mixed?  
.....
- List the combinations that do not result in a new colour. ....  
HINT: Mixing blue with blue i.e. BB does not give a new colour.
- What is the probability that the mixture does not result in a new colour? .....
- What is the probability that the mixture produces a new colour? .....
- Orange is a mixture of red and yellow. What is the probability that the new colour can be described as orange? .....

## Experiment

Do your own colour experiment. Make two hexagonal spinners. On each spinner mark three equal sections and label/colour them blue, yellow, red. Spin both spinners and record the results in the table.

|         | Tally | Total/<br>30 | Tally | Total/<br>60 | Tally | Total/<br>90 | Tally |  |  |  |  |  |
|---------|-------|--------------|-------|--------------|-------|--------------|-------|--|--|--|--|--|
| BB      |       |              |       |              |       |              |       |  |  |  |  |  |
| BY (YB) |       |              |       |              |       |              |       |  |  |  |  |  |
| BR (RB) |       |              |       |              |       |              |       |  |  |  |  |  |
| YY      |       |              |       |              |       |              |       |  |  |  |  |  |
| YR (RY) |       |              |       |              |       |              |       |  |  |  |  |  |
| RR      |       |              |       |              |       |              |       |  |  |  |  |  |

Remember a mixture of yellow and red makes orange.

Start by spinning your spinners 30 times. How many times did you get orange? .....

So orange came up ..... times.  
30

Do another 30 spins. Now count the number of times you got orange.

Orange came up ..... times.  
60

Look back at the **probability** of making orange (from Activity 2). What was it? .....

When you do an experiment your results will only be close to the probability if you carried out the experiment a large number of times (and, as far as this is concerned, 60 is small). Go on spinning your two spinners in batches of 30 and see if your results get closer to the expected probability.



### Review

Do you need more practice in sample space tables?

Yes ☐ No ☐

For more work on this, go to H1 (page 10) and E2 (page 11).

## Tree diagrams

Another event that has two possible outcomes is tossing a coin.

What are the possible outcomes when I toss a coin twice?

What is the probability of getting two heads?

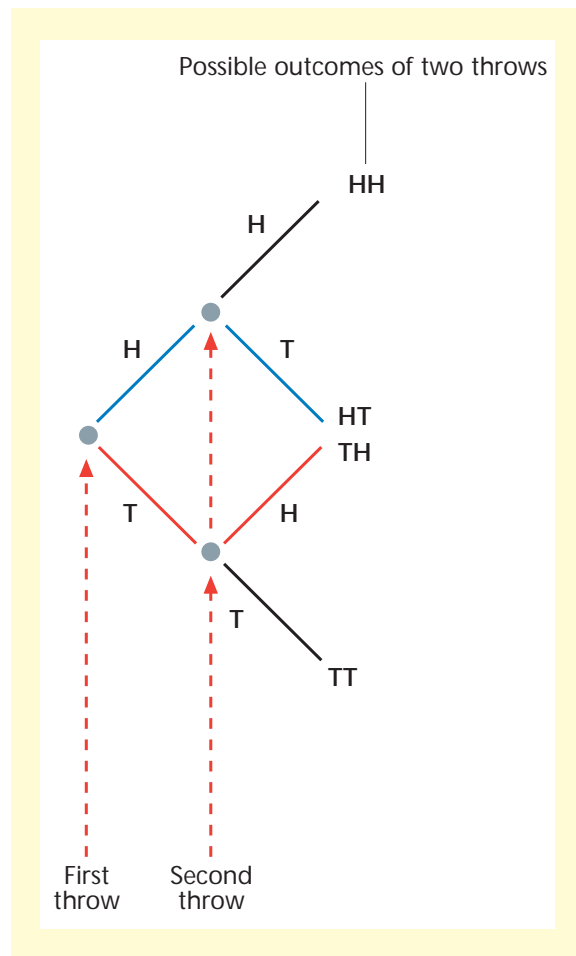
This is another way of showing the outcomes. It is called a tree diagram.

The toss of the coin is shown by the dots.

The two branches from the dots show the two possible outcomes, H (heads) or T (tails).

For example, follow the blue line. This shows that the first coin is a head and that the second coin is a tail. The outcome is HT, one head and one tail.

The red line also shows an outcome of one tail and one head, only this time the tail was thrown first, followed by the head (TH).



### Activity 3

- 1 How many possible outcomes of the two throws are there? .....
- 2 What are the chances of throwing two heads? Write the answer as a fraction.  
.....
- 3 What are the chances of throwing two tails? Write the answer as a fraction.  
.....
- 4 What are the chances of throwing one head and one tail? Write the answer as a fraction.  
.....

The number of possible outcomes of tossing a coin is the same as the number of possible outcomes for the sex of twins! There are two possible outcomes for each single event (head/tail, girl/boy), and four possible outcomes for the combined events.



### Activity 4

Draw a sample space table to show the possible outcomes of tossing a coin twice.



### Activity 5

Draw a tree diagram to show the possible outcomes of the sex of twins.

## Activity 6

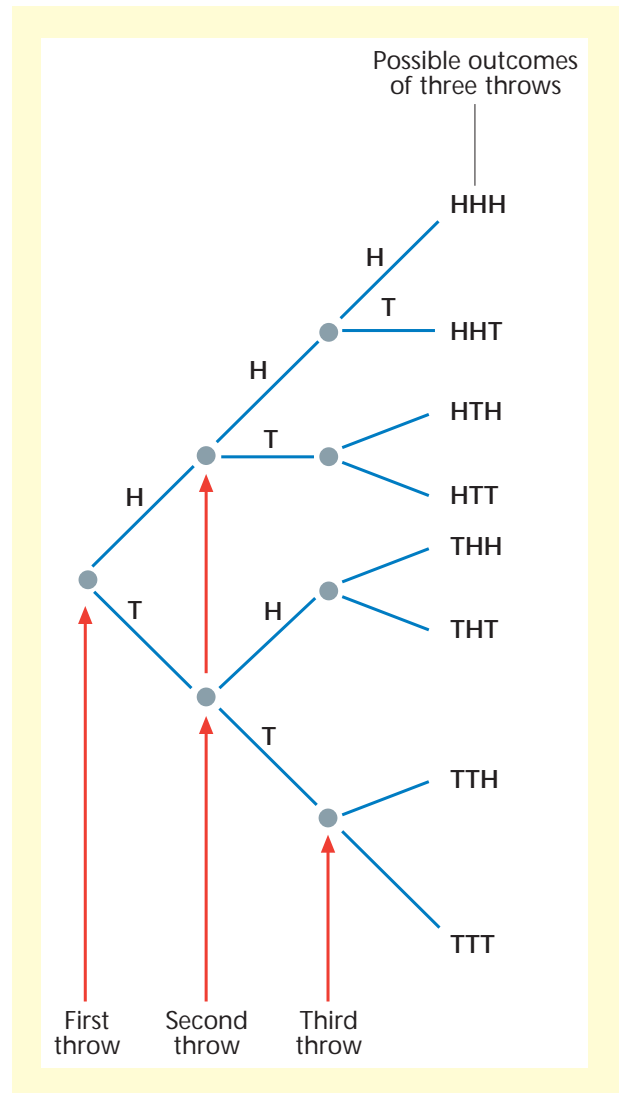
If I toss a coin three times, what are the chances of getting three heads? Are the chances better or worse or the same as getting two heads with two throws?

Look at the tree diagram.

Fill in the missing H (heads) and T (tails) on the branches for the third throw.

Answer these questions using the tree diagram

- 1 How many possible outcomes are there?  
.....
- 2 How many ways can I throw three heads?  
.....
- 3 What is the probability of throwing three heads?  
.....
- 4 Look back to Activity 3 to find the probability of throwing two heads with two tosses. It was  
.....
- 5 Do I have a better chance of throwing three heads with three tosses or two heads with two tosses?  
.....



## Activity 7

Write down the probabilities, as fractions, of throwing:

- 1 one head with one toss of a coin .....
- 2 two heads with two tosses of a coin .....
- 3 three heads with three tosses of a coin .....
- 4 Do you notice a pattern? .....
- 5 What do you think the probability will be of throwing four heads with four tosses?  
.....

## Talk about it

Discuss your answers with your group and with your teacher.

## Review

Do you need more practice in tree diagrams?

Yes ☐ No ☐

For more work on this, go to H2 (page 10) and E1 (page 11).

## What's the score?

If I throw two dice and add the scores, what is the most likely total score?

Look at the table. Along the top it shows the possible score on the first dice.

Down the side it shows the possible score on the second dice.

The coloured squares show the total score of the two dice. Complete the table.

|                      |   | score on first dice |   |   |   |   |    |
|----------------------|---|---------------------|---|---|---|---|----|
|                      |   | 1                   | 2 | 3 | 4 | 5 | 6  |
| score on second dice | 1 | 2                   | 3 | 4 | 5 | 6 | 7  |
|                      | 2 | 3                   | 4 |   |   |   |    |
|                      | 3 | 4                   |   |   | 7 |   |    |
|                      | 4 | 5                   | 6 |   |   | 9 | 10 |
|                      | 5 | 6                   |   |   | 9 |   |    |
|                      | 6 | 7                   | 8 |   |   |   |    |

### Remember

To simplify a fraction, divide the numerator (top) and denominator (bottom) the fraction by the same number.

e.g.  $\frac{6}{16} = \frac{6 \div 2}{16 \div 2} = \frac{3}{8}$

e.g.2  $\frac{25}{30} = \frac{25 \div 5}{30 \div 5} = \frac{5}{6}$



### Activity 8

Simplify your answers where possible. Give your answers to probability questions as fractions, unless otherwise stated.

- How many possible outcomes are there? (How many coloured squares are there?)  
.....
- Which total score appears most often in the table? .....
- What is the probability of throwing a total of 7? .....
- What is the probability of throwing a total of 2? .....
- Which other total score has the same probability as a score of 2? .....
- You are ..... times more likely to score a total of 7 than a total of 2 with two dice.

7 What is the probability of a total of **more than 8**.

.....

8 What is the probability of a total of **less than 4**.

.....

9 a What is the probability of the total score being **even**?

.....

b What is the probability of the total score being **odd**? .....

c Add your answers to parts **a** and **b** together.

.....

d Explain your answer to part c.

.....

.....

.....

10 List the outcomes that are prime numbers.

.....

11 Work out the probability of scoring a prime number.  
Give your answer as a fraction and as a decimal.

.....

12 Work out the probability that your score is not a prime number.  
Give your answer as a fraction and as a percentage.

.....

13 What is the probability of scoring a multiple of 5 a as a fraction? .....  
b as a decimal? .....  
c as a percentage? .....

### Remember

**Prime numbers** are numbers that have exactly two factors: the number itself and 1.

That is 2, 3, 5,...



### Review

Do you need more practice with probability tables?

Yes ☐ No ☐

For more work on this, go to E1 (page 11).

## Get on and do it!



'Scissors, Paper, Stone' is a traditional and entertaining game for two or more players. Scissors are shown by using two fingers as if about to cut. Paper is shown by holding the hand flat. Stone is shown by forming a fist. All players show their hand at the same time. Revealing the same shape is a draw. However, scissors cut (beats) paper, paper wraps (beats) stone, and stone blunts (beats) scissors.



### Activity 9

The possible outcomes of **two players** can be shown in a table. Complete the table. C means scissors, P means paper, S means stone.

|          |   | Player 1 |    |    |
|----------|---|----------|----|----|
|          |   | C        | P  | S  |
| Player 2 | C | CC       | PC | SC |
|          | P | CP       | PP |    |
|          | S | CS       |    |    |

- 1 How many possible outcomes are there for the results of each show? .....
- 2 How many outcomes are there where the players draw? .....
- 3 What is the probability that the players will draw? .....
- 4 What is the probability that Player 1 wins? .....
- 5 What is the probability that Player 1 does not win? .....

### Remember

#### Independent events

Two events are independent if one event does not affect the outcome of the other event e.g. tossing a coin and rolling a dice.

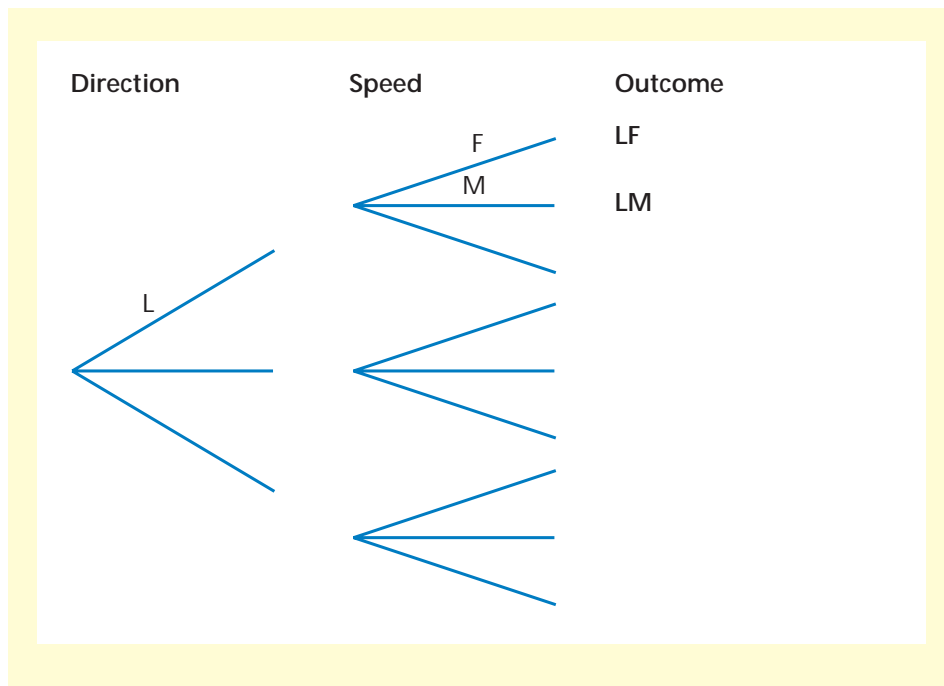


## Activity 10

Some tennis players like to practise with an automatic ball server. The machine can be set to vary the direction and the speed of the tennis ball. Balls are then served randomly at the player.

For beginners the directions are left (L), right (R) and centre (C), and the speeds are fast (F), medium (M) and slow (S). For example, a ball may be served at the player to the left and at a medium speed.

- 1 Complete the tree diagram to show the outcomes.



- 2 What is the probability that a player gets a slow ball served to the right?

.....

- 3 What is the probability that a player gets a ball served to the right?

.....

## Review

Do you need more practice with probability tables?

Yes ☐ No ☐

For more work on this, go to H2, (page 10) and E1 and E2 (page 11).

This work links to mini-projects M1 and M2 (page 12).

## Activity H1

Answer these questions about the black (B) and white (W) spinner.

- 1 What is the probability of getting black in one spin?

.....

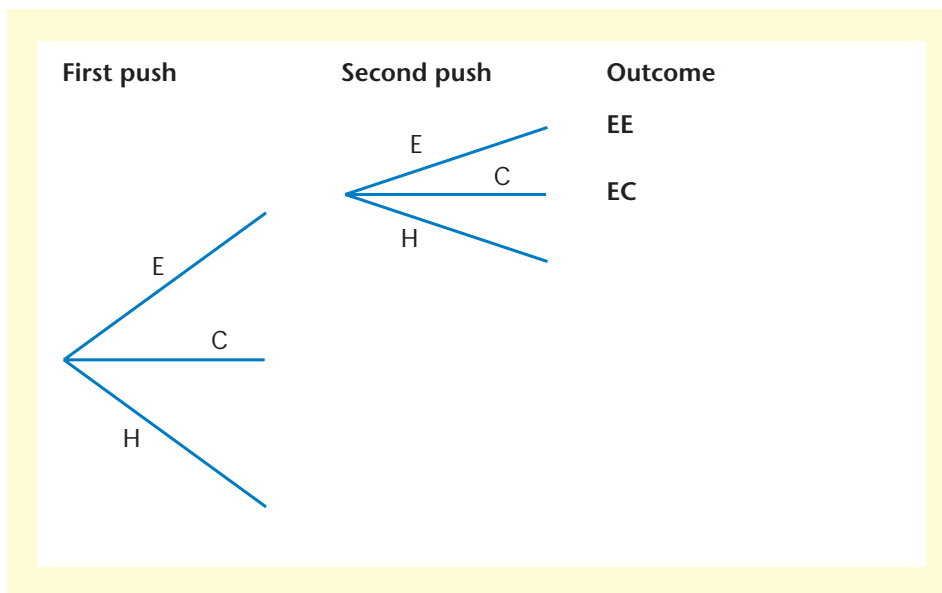
|             |   | First spin |   |
|-------------|---|------------|---|
|             |   | W          | B |
| Second spin | W |            |   |
|             | B |            |   |

- 2 What is the probability of getting different colours in two spins? .....



## Activity H2

Fred has a random button on his car radio. He has preset it with his favourite stations. At the push of a button he gets a station that he likes while he's driving. At the moment he has only preset three stations: Easy FM (E), ChatRadio (C) and HitUK (H). Complete the tree diagram for two pushes of the random button.



- 1 What is the probability Fred gets Easy FM (E) twice in a row?

.....

- 2 What is the probability Fred gets the same station twice in a row?

.....



## Extension



### Activity E1

There is a simple card game at the fair. There are two cards on a table face down: an ace and a king.

A player points to a card and it is turned over. If it is an ace you score 10 points. The king scores 5 points. The card is then turned back over and the cards mixed up. This is done three times.

Susie has a go. If she scores exactly 20 she gets her money back. If she scores exactly 30 she wins £1.



- 1 Draw a tree diagram to show the three outcomes of her three choices. Put the points score of each outcome. (Hint: use a full sheet of paper – the finished diagram may be large!)
- 2 What is the probability that Susie scores 15 points? .....
- 3 What is the probability that Susie wins £1? .....
- 4 Put a ring round the final outcomes on your diagram that show 20 points.
- 5 What is the probability that Susie gets her money back? .....



### Activity E2

Will makes a game for the fair. It uses two dice: one is a standard dice and the other has the numbers 5 to 10. The game asks that the players **multiply** the scores on the dice. Will needs to look at the possible outcomes to ensure that he will make a profit from the game.

- 1 Draw a sample space table to show all the outcomes.
- 2 What is the probability of scoring 25?  
.....
- 3 What is the probability of scoring 1?  
.....
- 4 What is the probability of scoring more than 50?  
Write your answer as a fraction.  
.....
- 5 What is the probability of scoring less than 20?  
Write your answer as a decimal.  
.....
- 6 What is the probability of scoring a prime number? Write your answer as a fraction.  
.....

#### Tip

To help you with the answers make your sample space table like this.

| (×) | 1 | 2  | 3  |
|-----|---|----|----|
| 5   | 5 | 10 | 15 |
| 6   | 6 | 12 |    |
| 7   | 7 |    |    |



# Mini-projects



## Activity M1

Play the game 'Paper, Scissors, Stone' with another person. Play it nine times. Record your answers.

Compare your results with the table in Activity 9. What do you notice?

Play the game 18, 27, 36 times. What do you notice?



## Activity M2

Roll two dice 36 times and record the number of times you score a total of 7.

Look back to Activity 8. How many times would you expect to score 7 with 36 throws? 72 throws? How close was your result?

How many times do you have to roll the dice to make your results close to the probability?



## Activity M3

Obtain a bus or train timetable for your local area.

Choose a particular bus or train service. Choose a stop/station.

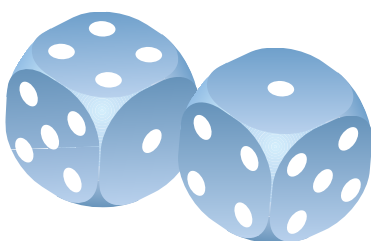
Use the timetable to record whether the bus or train is late/on time/early over a number of occasions.

You will need to decide **exactly** what 'on time' means and how you will know this. You will also need to decide on how many 'occasions' you are going to record this information.

To what extent would you feel confident in using this information to determine the time you turned up to catch this bus or train in the future?

What is the probability that the bus/train will be:

- late
- on time
- early?



# Check it

## Activity C1

1 What is the probability of scoring 5 or more in one throw of a standard dice?

.....

2 What is the probability of throwing two heads when a coin is tossed twice?

.....

In the space below, draw a sample space diagram to help you.

## Activity C2

The table shows the possible outcomes of throwing two dice. The coloured squares show the two numbers on the dice. Complete the sample space table.

|                      |   | Score on first dice |      |      |      |      |      |
|----------------------|---|---------------------|------|------|------|------|------|
|                      |   | 1                   | 2    | 3    | 4    | 5    | 6    |
| Score on second dice | 1 | 1, 1                | 2, 1 | 3, 1 | 4, 1 | 5, 1 | 6, 1 |
|                      | 2 | 1, 2                | 2, 2 |      |      |      |      |
|                      | 3 | 1, 3                |      |      |      |      |      |
|                      | 4 | 1, 4                |      |      |      |      |      |
|                      | 5 | 1, 5                |      |      |      |      |      |
|                      | 6 | 1, 6                |      |      |      |      |      |



Use the table on the previous page to answer these questions.

- 1 How many possible outcomes are there? .....
- 2 What is the probability of throwing a double (both numbers the same)?  
.....
- 3 Have you got a better chance of throwing double 1 or double 6? Use the probabilities to explain your answer.  
.....

- 4 Choose a number from 1 to 6. If you throw only one dice, what are the chances of throwing this number?  
.....

Look at the outcomes in the table. Mark every square that shows the number you chose.

- 5 If you throw two dice, what are the chances of your number coming up on at least one dice?  
.....

Is this a better or worse chance than with one die? .....

Use probabilities to explain your answer.  
.....

## How am I doing?

Now look back at the skills listed on page 1.  
Then complete the sentences below.

I am confident with

.....  
.....

I need more practice with

.....

Date .....

## Activity 1

1 4

2 a quarter,  $\frac{1}{4}$

3  $\frac{1}{4}$

4  $\frac{2}{4} = \frac{1}{2}$

Probability of twins the same sex is  $\frac{2}{4}$  which is the same as  $\frac{1}{2}$ .

## Activity 2

|            | Blue (B) | Yellow (Y) | Red (R) |
|------------|----------|------------|---------|
| Blue (B)   | BB       | YB         | RB      |
| Yellow (Y) | BY       | YY         | RY      |
| Red (R)    | BR       | YR         | RR      |

1 9

2 BB, YY, RR

3  $\frac{3}{9} = \frac{1}{3}$

4  $\frac{6}{9} = \frac{2}{3}$

5  $\frac{2}{9}$

## Activity 3

1 4

2  $\frac{1}{4}$

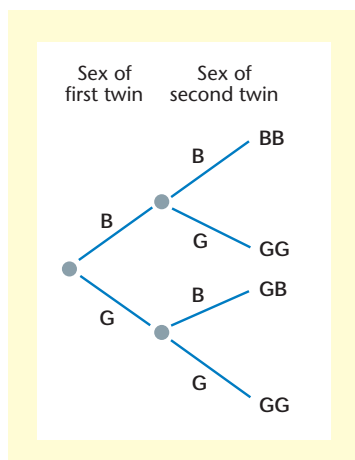
3  $\frac{1}{4}$

4  $\frac{2}{4} = \frac{1}{2}$

## Activity 4

|   | H  | T  |
|---|----|----|
| H | HH | TH |
| T | HT | TT |

## Activity 5



## Activity 6

1 8

2 1 way (HHH)

3  $\frac{1}{8}$

4  $\frac{1}{4}$

5 Two heads with two tosses

## Activity 7

1  $\frac{1}{2}$ , 2  $\frac{1}{4}$ , 3  $\frac{1}{8}$

4 Check with your teacher.

5  $\frac{1}{16}$

## Activity 8

|                      |   | Score on first dice |   |   |    |    |    |
|----------------------|---|---------------------|---|---|----|----|----|
|                      |   | 1                   | 2 | 3 | 4  | 5  | 6  |
| Score on second dice | 1 | 2                   | 3 | 4 | 5  | 6  | 7  |
|                      | 2 | 3                   | 4 | 5 | 6  | 7  | 8  |
|                      | 3 | 4                   | 5 | 6 | 7  | 8  | 9  |
|                      | 4 | 5                   | 6 | 7 | 8  | 9  | 10 |
|                      | 5 | 6                   | 7 | 8 | 9  | 10 | 11 |
|                      | 6 | 7                   | 8 | 9 | 10 | 11 | 12 |

1 36

2 7

3  $\frac{6}{36} = \frac{1}{6}$

4  $\frac{1}{36}$

5 12

6 You are six times more likely to score a total of 7 than a total of 2.

7  $\frac{10}{36} = \frac{5}{18}$

8  $\frac{3}{36} = \frac{1}{12}$

9 a  $\frac{1}{2}$  b  $\frac{1}{2}$  c 1 d Check your answer with your teacher.

10 2, 3, 5, 7, 11

11  $P(\text{prime}) = \frac{15}{36} = 0.417$

12  $P(\text{not prime}) = \frac{21}{36} = \frac{7}{12} = 58\%$

13  $P(\text{multiple 5}) = \frac{7}{36} = 0.194 = 19\%$

## Activity 9

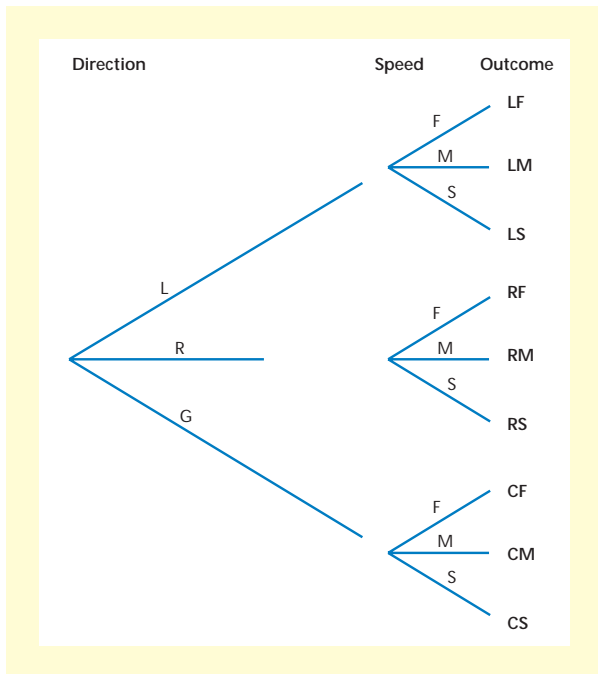
|          |   | Player 1 |    |    |
|----------|---|----------|----|----|
|          |   | C        | P  | S  |
| Player 2 | C | CC       | PC | SC |
|          | P | CP       | PP | SP |
|          | S | CS       | PS | SS |



- 1 9
- 2 3
- 3  $\frac{3}{9} = \frac{1}{3}$  (CC, PP, SS)
- 4  $\frac{3}{9} = \frac{1}{3}$  (CP, PS, SC)
- 5  $\frac{6}{9} = \frac{2}{3}$  (CC, PP, SS, PC, SP, CS)

### Activity 10

1



- 2  $\frac{1}{9}$
- 3  $\frac{3}{9} = \frac{1}{3}$

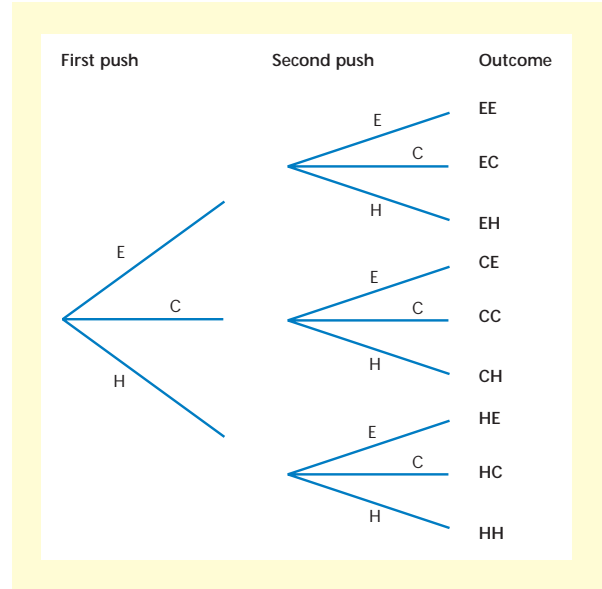
### Help

#### Activity H1

|          |   | 1st spin |    |
|----------|---|----------|----|
|          |   | W        | B  |
| 2nd spin | W | WW       | BW |
|          | B | WB       | BB |

- 1  $\frac{1}{2}$
- 2  $\frac{1}{2}$

### Activity H2

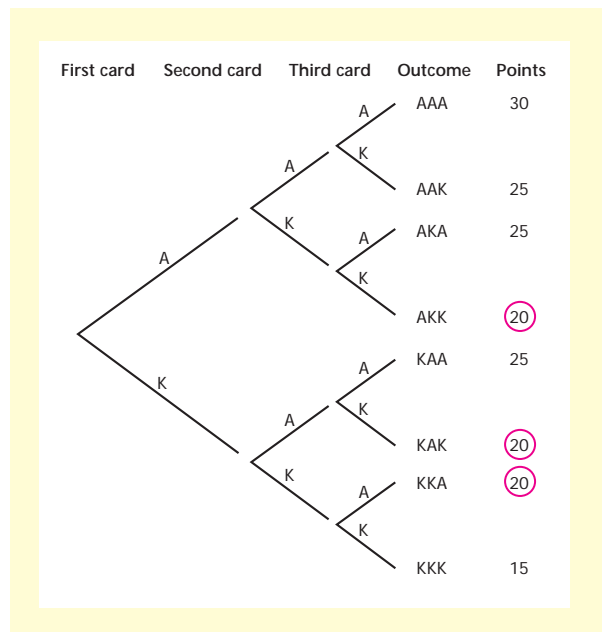


- 1  $\frac{1}{9}$
- 2  $\frac{3}{9} = \frac{1}{3}$

### Extension

#### Activity E1

1



- 2  $\frac{1}{8}$
- 3  $\frac{1}{8}$
- 4 Outcomes AKK, KAK, KKA circled.
- 5  $\frac{3}{8}$





**E2**

1

|            |    | Standard dice |    |    |    |    |    |
|------------|----|---------------|----|----|----|----|----|
|            |    | 1             | 2  | 3  | 4  | 5  | 6  |
| Other dice | 5  | 5             | 10 | 15 | 20 | 25 | 30 |
|            | 6  | 6             | 12 | 18 | 24 | 30 | 36 |
|            | 7  | 7             | 14 | 21 | 28 | 35 | 42 |
|            | 8  | 8             | 16 | 24 | 32 | 40 | 48 |
|            | 9  | 9             | 18 | 27 | 36 | 45 | 54 |
|            | 10 | 10            | 20 | 30 | 40 | 50 | 60 |

2  $\frac{1}{36}$

3 0

4  $\frac{2}{36} = \frac{1}{18}$

5  $\frac{13}{36} = 0.36$

6  $\frac{2}{36} = \frac{1}{18}$

### Mini-projects

Check your work with your teacher.

### Check it

#### Activity C1

1  $\frac{2}{6} = \frac{1}{3}$

2  $\frac{1}{4}$

Check your sample space table with your teacher.

**C2**

|                      |   | Score on first dice |      |      |      |      |      |
|----------------------|---|---------------------|------|------|------|------|------|
|                      |   | 1                   | 2    | 3    | 4    | 5    | 6    |
| Score on second dice | 1 | 1, 1                | 2, 1 | 3, 1 | 4, 1 | 5, 1 | 6, 1 |
|                      | 2 | 1, 2                | 2, 2 | 3, 2 | 4, 2 | 5, 2 | 6, 2 |
|                      | 3 | 1, 3                | 2, 3 | 3, 3 | 4, 3 | 5, 3 | 6, 3 |
|                      | 4 | 1, 4                | 2, 4 | 3, 4 | 4, 4 | 5, 4 | 6, 4 |
|                      | 5 | 1, 5                | 2, 5 | 3, 5 | 4, 5 | 5, 5 | 6, 5 |
|                      | 6 | 1, 6                | 2, 6 | 3, 6 | 4, 6 | 5, 6 | 6, 6 |

1 36

2  $\frac{6}{36} = \frac{1}{6}$

3  $P(1,1) = \frac{1}{36}$ ,  $P(6,6) = \frac{1}{36}$ , so no better chance

4  $\frac{1}{6}$

5  $\frac{11}{36}$  – better chance than on one dice, which is  $\frac{1}{6}$  or  $\frac{6}{36}$

Check your explanation with your teacher.