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CENTRES FOR EXCELLENCE IN TEACHER TRAINING
The Association of Centres for Excellence in Teacher Training

Teaching New GCSE Maths Specifications

Ratio, Proportion & Algebra

Participant Handbook

Contents list

Introduction and aims for the day

Welcome to the 'Teaching new GCSE Maths specifications - Ratio, Proportion & Algebra' day.

Aims of the programme

The highly successful Maths Enhancement Programme set a new standard in the training of Maths teachers in the Post 16 Sector in England. This subsequent set of three single days aims to use the same principles to provide development for teachers of Foundation Level GCSE, particularly those who have only to date taught Functional Skills.

However, there is debate and development to be had for all participants in each of the three days which we hope will result in confident and capable deliverers of the new GCSE subject content & assessment objectives for teaching from 2015.

Aims for Ratio, Proportion & Algebra day

- For participants to identify innovative and activity based ways of working with Post 16 learners.
- To begin to self-assess and reflect on personal Maths skills.
- To consider the factors that might engage Post 16 learners with Maths and develop their enthusiasm for the subject.
- To develop connections between different topics in maths, and between maths and everyday situations.
- To discuss and debate new style examination questions in the field of ratio, proportion and algebra.

Making the most of this handbook

One day courses are renowned for inspiring and enthusing teachers but this can only have a long term effect if you develop plans to change your practice that are sustainable. This handbook gives you the opportunity to consider the activities you experience and after each one to make notes on what changes you can make to your practice that will engage your own learners.

Don't be afraid to criticise the activities if you feel that is necessary. However, analyse your thoughts and go away with notes on how you will use the materials differently to best effect for the benefit of your own learners.

Create your action plan and then implement it!!

Starter activity: Mind Reading

(Adapted from NRICH: http://nrich.maths.org/933)

Aims:

- To engage delegates on arrival.
- To start delegates thinking algebraically.

Resources:

Adapted from NRICH: http://nrich.maths.org/933 & http://nrich.maths.org/2017

Activity:

- Look at the slide displayed on arrival.
- Does it always work? Try a few different starting numbers.
- Can you work out how this 'mind reading' is done? Discuss the problem with someone else on your table.
- Can you explain how it's done to someone else?

If you finish this task, ask the trainer for the **extension activity**.

Session 1: At the chip shop

Aims:

To help delegates to:

- Develop/consolidate their understanding of substitution, simplification & factorisation.
- Make connections between algebra and a familiar everyday context.

Resources:

Adapted from MEI: http://www.mei.org.uk/files/conference15/K6-Sue-Steve-pdf.pdf

Activity:

- This session uses the context of a chip shop to make connections between algebra and a real-life context. It also introduces some of the core processes & terminology of algebra.
- Follow through the slides and activities, working with a partner or on your own, as you prefer.

Individual Review:

What did you learn from the activity?

What was it about the activity that helped you learn?

Does the activity differentiate to meet the needs of a range of learners?

How might you adapt the activity with your learners?

Session 2: Interpreting algebraic expressions

Aims:

To help delegates to:

- Understand the equivalence of the word, symbol and area representations of algebraic expressions.
- Recognise the order of operations.
- Recognise equivalent expressions.
- Understand the distributive law.
- Explore factorising and multiplying out quadratic expressions.

Resources:

Adapted from *Improving Learning in Mathematics:* http://www.nationalstemcentre.org.uk/elibrary/resource/1998/interpreting-algebraic-expressions-a1

Activity:

- Part 1 of the activity involves listening to the trainer's verbal descriptions of arithmetic processes, which you are asked to write down as algebraic expressions on your mini-whiteboard.
- Following this, you will work in a group and take turns to match algebraic expressions with their explanations in words. Make sure that you explain your reasoning to others in your group, and challenge your peers if you don't agree or don't understanding something. *Note: not all the cards match!*
- After a short plenary discussion, the final part of the activity involves you matching the expressions shown as area diagrams.

Individual Review:

What did you learn from the activity?

What was it about the activity that helped you learn?

Does the activity differentiate to meet the needs of a range of learners?

How might you adapt the activity with your learners?

Session 3: Solving linear equations

Aims:

To help delegates to:

- Form and solve linear equations involving factorising and using the distributive law.
- Use variables to represent quantities in real-world and mathematical problems.
- Solve word problems leading to equations of the form px + q = r and p(x + q) = r.

Resources:

Adapted from *Mathematics Assessment Project*:
http://map.mathshell.org/lessons.php?unit=7220&collection=8

Activity:

- Part one of the activity involves you looking at an algebra 'story' and matching the appropriate equations to the story.
- Following this, you are asked to work individually to write equations that describe different stories.
- Next, you work in small group to match the stories with cards containing different equations, drawing on the individual thinking that you have done already.
- Finally, you work in small groups to solve each of the equations, using another set of cards to show the steps, and display your final work as a poster.

Individual Review:

What did you learn from the activity?

What was it about the activity that helped you learn?

Does the activity differentiate to meet the needs of a range of learners?

How might you adapt the activity with your learners?

Session 4: Developing proportional reasoning

Aims:

To help delegates to:

- Reflect on the range of contexts in which ratio & proportion are used.
- Examine proportional problems and appreciate their multiplicative structure.
- Distinguish between direct proportion and other functional relationships.
- Solve proportionality problems by efficient methods.

Resources:

Adapted from *Mathematics Assessment Project*:
http://map.mathshell.org/lessons.php?unit=7215&collection=8

Activity:

- Initially, you are asked to consider the meaning of a ratio, and the different contexts in which ratios are used – in GCSE maths, and across vocational & everyday contexts.
- The main activity starts with a proportional problem with values missing. The task is for you each to suggest pairs of values that would fit in with the scenario; this is used to explore different methods of working with proportion, and the links between them. From this, the properties of direct proportion are derived, and what direct proportion looks like when sketched on a graph.
- Next, you will work in a small group to substitute values within a range of different everyday and mathematical scenarios, and decide in each case whether the scenario is directly proportional or not.
- Finally, groups will write their own questions to swap with other groups to solve, and then check their responses.

Individual Review:

What did you learn from the activity?

What was it about the activity that helped you learn?

Does the activity differentiate to meet the needs of a range of learners?

How might you adapt the activity with your learners?

Session 5: Exploring straight line graphs

Aims:

To help delegates to:

- Make connections between proportional situations, straight line graphs and linear equations.
- Interpret the general equation for straight line graphs: $y = mx + c$.
- Explore how varying a straight line equation affects the gradient and y-intercept.
- Relate straight line graphs and equations to everyday contexts.

Resources:

It is recommended that participants bring a laptop or tablet with a graphing application, such as GeoGebra: https://www.geogebra.org pre-installed (1 between 3 participants should be sufficient). Alternatively, websites such as 'Maths is Fun' have built in apps - see https://www.mathsisfun.com/data/straight_line_graph.html.

Activity:

- The first part of the activity involves you plotting graphs and producing equations to represent proportional and non-proportional everyday situations. This is used to derive a general equation for a straight-line graph.
- Next you have the opportunity to experiment with graphing software to investigate how changing the equation affects the appearance of the graph, and to identify the relationship between the two.
- Finally, you are asked work in groups to apply this knowledge and match cards containing corresponding equations, graphs and everyday contexts.

Individual Review:

What did you learn from the activity?

What was it about the activity that helped you learn?

Does the activity differentiate to meet the needs of a range of learners?

How might you adapt the activity with your learners?

Session 6: Comparing approaches

Aims:

To help delegates to

- Interpret a situation and represent the variables mathematically.
- Select appropriate mathematical methods to use.
- Interpret and evaluate the data generated and identify the break-even point, checking it for confirmation.

Resources:

Adapted from *Mathematics Assessment Project*:
http://map.mathshell.org/lessons.php?unit=8200&collection=8

Activity:

- This activity starts with you working in groups to solve a problem about the cheapest way of purchasing football shirts, using any method you wish.
- Afterwards, you are asked to discuss a series of sample responses to the same problem, and to compare and evaluate the different approaches.

Individual Review:

What did you learn from the activity?

What was it about the activity that helped you learn?

Does the activity differentiate to meet the needs of a range of learners?

How might you adapt the activity with your learners?

Session 7: Sequences

Aims:

To help delegates to:

- Understand the concepts, processes & notation of arithmetic sequences.
- Relate arithmetic sequences to linear equations & straight line graphs.

Resources:

These have been partly produced for this module, and partly adapted from a previous NCETM module: 'Taking GCSE deeper: algebra & statistics'.

Activity:

- The session starts by asking you to produce a table of values derived from a linear equation. This is used to make the link with a pictorial form of the same table of values, and then to a linear (or arithmetic) sequence – including the convention for writing these and expressing the n^{th} term.
- Following this, you are asked to work collaboratively to analyse other sequences with gaps in them, and in each case to specify the expression for the n^{th} term.

Individual Review:

What did you learn from the activity?

What was it about the activity that helped you learn?

Does the activity differentiate to meet the needs of a range of learners?

How might you adapt the activity with your learners?

Session 8: Exploring exam questions

Aims:

To enable delegates to:

- Explore the demand of GCSE exam questions with regard to ratio, proportion & algebra.
- Work collaboratively to apply maths skills in solving problems.

Resources:

- Selection of ratio, proportion & algebra questions from specimen papers for the new GCSE maths specifications (2016 examination) – foundation tier (available from awarding body websites)
- The new specifications can be downloaded from: https://www.gov.uk/government/publications/gcse-mathematics-subject-content-and-assessment-objectives

Activity:

- The activity involves you working collaboratively with others to explore and solve some GCSE ratio, proportion & algebra questions taken from awarding body specimen papers for the new (2016) GCSE specifications.
- Solutions to questions will be shared, along with any problems experienced. Ideas for ways of working with exam questions will also be shared.

Individual Review:

What did you learn from the activity?

What was it about the activity that helped you learn?

Does the activity differentiate to meet the needs of a range of learners?

How might you adapt the activity with your learners?

Glossary of terms used in GCSE ratio, proportion & algebra:

Arithmetic progression	A sequence of numbers in which terms are generated by adding or subtracting a constant amount to the preceding term. Examples: 3, 11, 19, 27, 35 … where 8 is added; 4, -1, -6, -11 ... where 5 is subtracted.
Binomial	A binomial expression is an expression with two terms, e.g. $3x + 4$
Coefficient	The number in front of an algebraic symbol. For example the coefficient of $5x$ is 5.
Constant	A number (or term) in an expression that does not vary. E.g. in the expression $3x + 6$, the 3 and 6 are constants, where x is a variable.
Direct proportion	The relationship between quantities whose ratio is constant
Equation	A mathematical statement showing that two expressions are equal. The expressions are linked with the symbol = Examples: $7 - 2 = 4 + 1$ $4x = 3$ $x^2 - 2x + 1 = 0$
Evaluate	Find the value of a numerical or an algebraic expression. e.g. Evaluate $x^2 - 3$ when $x = 2$ by substituting this value for x and calculating, $2^2 - 3 = (2 \times 2) - 3 = 4 - 3 = 1$
Expand	To multiply out brackets in an expression. For example, $2(3x + 7) = 6x + 14$.
Exponent	Also known as index, a number, positioned above and to the right of another, indicating repeated multiplication. Examples: n^2 indicates $n \times n$; and n^5 indicates $n \times n \times n \times n \times n$.
Expression	A collection of terms which can contain constants (numbers) and variables (letters) e.g. $4pq - q + 7$
Factor	The factors of an expression in algebra are two (or more) other expressions which can be multiplied together to produce the original expression, e.g. 2, x and (2x-3) are both factors of the expression $4x^2 - 6x$
Factorise	To put an expression into brackets by taking out a common factor. For example, $20x + 15y = 5(4x + 3y)$.
Formula	An equation linking two or more physical variables, so that when one or more values are known, the value of one particular quantity can be found (e.g. In $A=\pi r^2$, if r is known, A can be found)
Function machine	A type of flow diagram intended to make clear the order in which operations must be carried out to obtain a result. e.g. $y = 3x^2 + 5$ can be shown by the function machine: 

Geometric progression	A sequence of numbers in which terms are generated by multiplying or dividing the preceding term by a constant amount. Examples: 2, 6, 18, 54 ... where the previous term is multiplied by 3; 24, 12, 6, 3 … where the previous term is divided by 2.
Gradient	A measure of how steep a line is. Found by dividing the distance up by the distance across.
Identity	An equation that holds for all values of the variables. The symbol ≡ is used. Example: $a^2 - b^2 \equiv (a + b)(a - b)$.
Indices	Another name for powers such as ² or ³.
Indirect proportion	Where one value increases as the other value decreases
Inequalities	Statements such as $a \neq b$, $a \leq b$ or $a > b$ are inequalities.
Like terms	Terms that are identical in respect of their variables, such as the pairs 3x and 5x, or $7x^2y$ and x^2y.
Linear equation	An equation where the highest power is 1, e.g. y = 3x + 2. Such an equation can be represented graphically by a straight line.
Root	A root of an equation is a value that will satisfy the equation, e.g. $x^2 – 8x + 15 = 0$ has two roots: x = 3 or 5.
Ratio	The relationship between two amounts, showing the number of times one value contains or is contained within the other. Example: In a recipe for pastry fat and flour are mixed in the ratio 1 : 2 which means that the fat used is half the amount of the flour.
Quadratic equations	An equation where the highest power is two. For example $x^2 + 4x + 6 = 0$ is a quadratic equation.
Sequence	A succession of terms formed according to a rule. There is a definite relation between one term and the next or between each term and its position in the sequence. Example: 1, 4, 9, 16, 25 …
Simplify	Write as simply as possible by gathering like terms, e.g. e +7e = 8e
Simultaneous equations	Two linear equations that apply simultaneously to given variables. The solution to the simultaneous equations is the pair of values for the variables that satisfies both equations. The graphical solution to simultaneous equations is a point where the lines representing the equations intersect.
Substitute	Replacing a variable with a known value (or part of an expression), e.g. Substitute the value x = 2 into the equation y = 4x – 5, giving y = (4 x 2) – 5.
Term	A number, variable or combination of both which forms part of an expression. E.g. the expression 5x – 3y + 4 has a total of 3 terms; 5x, -3y and 4.

Transpose	To transpose an equation or formula is to rearrange it to produce an equivalent version. E.g. the equation 4x + y = 5 can be transposed to y = 5 – 4x.
Trial & improvement	A method of looking for a solution in which a guessed-at value is put into an equation, and on the basis of the error found, a better guess is made, and so on until the level of accuracy required is obtained.
Variable	A value that can vary – often denoted by a letter (e.g. x or y)
Y-intercept	The value of the y-coordinate when a graph crosses the y-axis.

A full glossary of terms for key stages 1 to 4 can be downloaded from:
http://www.nationalstemcentre.org.uk/elibrary/resource/4784/mathematics-glossary-for-teachers-in-key-stages-one-to-four