The Education & Training Foundation

acett CENTRES FOR EXCELLENCE IN TEACHER TRAINING
The Association of Centres for Excellence in Teacher Training

Teaching New GCSE Maths Specifications

Ratio, Proportion & Algebra

Trainer Handbook

On arrival

Prepare the PowerPoint and projector. Display **slide 1**, 'While you're waiting …

Suggested timings are:

09:30-10:00	Refreshments and starter
10:00-10:15	Welcome and introductions
10:15-10:35	Session 1: At the chip shop – algebra in an everyday context
10:35-11:10	Session 2: Interpreting algebraic equations
11:10-11:20	Plenary
11:20-11:35	*Break*
11:35-12:15	Session 3: Solving linear equations
12:15-12:50	Session 4: Developing proportional reasoning
12:50-13:00	Plenary
13:00-13:30	*Lunch*
13:30-14:15	Session 5: Exploring straight-line graphs
14:10-14:40	Session 6: Comparing approaches
14:49-14:50	Plenary
14:50-15:00	*Break*
15:00-15:25	Session 7: Sequences
15:25-15:50	Session 8: Exploring algebra exam questions
15:50-16:00	Review and evaluation

Note: Identify beforehand what you could miss out/shorten if you find that something in a session has taken much longer than anticipated. The participants can be told if you have omitted an activity and that they can still download the activities.

Starter activity: Mind Reading

(Adapted from NRICH: http://nrich.maths.org/933)

Aims:

- To engage delegates on arrival.
- To start delegates thinking algebraically.

Resources:

- HO1: Mind Reading (extension)

Timing: Flexible – participants complete activity as they arrive.

Activity

- **Slide 1:** on arrival, and following registration, direct delegates to slide and suggest that they might want to work with someone else.
- If delegates struggle to get started, suggest they see what happens with several different starting numbers. Can they work out what's happening?
- When delegates have tried a few different numbers, ask them if they can use algebra to explain what's happening.
- Don't take feedback at this stage – the activity will be returned to later in the session.
- Those who complete the first activity can be given an extension – HO1: Mind reading. This is a slightly more difficult example.

Feedback:

Don't take feedback at this stage – leave the activity open. Explain that we will return to the activity later in the session.

Session 1: At the chip shop

(adapted from: MEI http://www.mei.org.uk/files/conference15/K6-Sue-Steve-pdf.pdf)

Aims:

To help delegates to:

- Develop/consolidate their understanding of substitution, simplification and factorisation.
- Make connections between algebra and a familiar everyday context.

Resources:

- HO2: At the chip shop
- Mini-whiteboards, pens and erasers

Timing: approx. 20-25 min (according to audience). You may be able to go through the activity quite quickly if people are familiar with the terms and concepts, but make sure everyone is on board.

Activity:

(NB: It is important that you constantly emphasise the chip shop context throughout the activity.)

- **Slide 5:** Ask participants to briefly discuss the meaning of each of term in pairs. Check that everyone has some idea of what is meant by each, though clarify that we are now going to look at each.
- **Slide 6:** Introduce the context of working in a chip shop – this will be the scenario for introducing algebra.
- **Slide 7:** Introduce the 2 people who work at the chip shop – Azim (left) and Jane (right). Ask participants to write down an algebraic expression to show what Jane is saying. (Check responses). Ask participants to write down an algebraic expression to show what Azim is saying. (Check responses). Are they the same? Why? Point out that these two algebraic ***expressions*** are known as ***identities*** because they are always the same.
- **Slide 8:** Make link with people at work, college, etc. placing orders for a group of people. Ask participants to write the order as an algebraic expression on mini-whiteboards (check responses)

- **Slide 9-11:** Nominate someone to explain what 1 and 2 mean in everyday language. Ask rest of group if they agree. Highlight that this is ***substitution***. Provide HO2, and ask delegates to work in pairs; they should use the menu to work out the cost of the orders, and write their answers on their mini-whiteboards. Nominate someone to explain why the two orders cost the same (use slide 10 to clarify if needed). Highlight that what they have just done is known as ***simplification***. If delegates need further practice, direct them to the 3 questions on slide 11 (also on HO2). Ask to work in pairs and check answers with others on their table.
- **Slide 12-13:** Emphasise the context again and check understanding of slide 12. Use mini-whiteboards to check responses to slide 13. Highlight key point about subtracting bracketed terms.
- **Slide 14-15:** Discuss slide 14, again emphasising the context, and how the order can be divided into 4 bags – each containing 3 fish and 4 chips. Ask to write on mini-whiteboards what this would look like algebraically. Highlight that this is ***factorisation***. If required, ask participants to try to factorise expressions on slide 15 and HO2.
- Check that everyone is now clear about the meaning of the 3 processes: *substitute, simply, factorise.*
- **Slide 16:** Highlight the top line of the table, and then ask delegates to complete the remainder of the table on HO2. Check answers with others on their tables and take any questions.

Session 2: Interpreting algebraic expressions

(Adapted from: *Improving Learning in Mathematics* http://www.nationalstemcentre.org.uk/elibrary/resource/1998/interpreting-algebraic-expressions-a1)

Aims:

To help delegates to:

- Understand the equivalence of the word, symbol, table and area representations of algebraic expressions.
- Recognise the order of operations.
- Recognise equivalent expressions.
- Understand the distributive law.
- Explore factorising and multiplying out quadratic expressions.

Resources:

- Mini-whiteboards, pens and erasers
- From Improving Learning in Mathematics, Session A1
    - Activity A1 Instruction sheet
    - Card set A – Algebraic expressions
    - Card set B – Expressions in words
    - Card set C – Tables of numbers *(NB Extension only)*
    - Card set D – Areas of shapes

Timing: approximately 35 min

Activity:

Whole group questioning (slide 17)

- Using mini-whiteboards ask the participants some or all the questions on slide 17.

Collaborative group work: matching algebraic expressions with explanations (slide 18)

- Arrange the participants in groups of 3 or 4 and give each group
    - Card Set A – *Algebraic expressions*
    - Card Set B – *Explanations in words*
- Ask participants to take turns at matching cards explaining their reasons for each matching. Point out that some cards are missing. Participants will need to make these extra cards themselves.
- The activity is designed to help participants interpret the symbols and realise that the symbolism defines the order of operations. Some participants may notice that some

expressions are equivalent, e.g. 2(n + 3) and 2n + 6. Do not comment on this at this stage.

Extension: Provide early finishers with Card set C Tables of numbers to match with card sets A and B.

- Plenary questions to prompt discussion:
  - Can anyone suggest reasons why different expressions appear to give the same answer?
  - Which ones are the same?
- Volunteers may like to offer suggestions on the board.
- Leave 'open' the question of why expressions are equivalent. The next part of the session will take these ideas further, so they do not need to be resolved at this point.

Collaborative group work: matching algebraic expressions with area diagrams (slide 19)

- Use the same groupings of 2 or 3 as chosen for Activity 2.
- Hand out the Areas of Shapes cards (Set D) and the Expressions cards (Set A). Ask participants to match these.

Extension: Ask groups that have completed the task to make new card matches of their own.

Feedback and return to starter activity (slide 20)

- Take feedback on the activity and briefly discuss learning points emerging from the activity.
- Ask delegates to discuss slide 20 in pairs, then show responses on mini-whiteboards.
- Highlight the similarity between C and the starter activity, and ask for a volunteer to discuss how the expression is arrived at.
- If time permits, return to the starter activity and discuss/demonstrate how the 'Mind reader' expression can be built up in steps.

Reflection (slide 21)

- Ask delegates to reflect on the activities carried out during sessions 1 and 2, in line with slide 21, and discuss in table groups. Invite them to make record their individual reflections in their handbooks.
- Take brief plenary feedback from each table.

Session 3: Solving linear equations

(Adapted from: Mathematics Assessment Project http://map.mathshell.org)

Aims:

To help delegates to:

- Form and solve linear equations involving factorising and using the distributive law.
- Use variables to represent quantities in a real-world or mathematical problems.
- Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$.

Resources:

- Mini-whiteboards, pens and erasers
- Flipchart sheets and markers
- Mathematics Assessment Project: Solving Linear Equations (downloadable PDF document)
    - Full instructions
    - Card set: Stories (not cut up) (S-3)
    - Card set: Stories (cut up) (S-3)
    - Card set: Equations (E1-E6) (S-4)
    - Card set: Steps to solving (S-5)

Timing: approximately 40 min

Activity:

- **Slide 22:** Highlight that we are now going to look at **equations**. Check that everyone is clear about what an equation is, and how it differs from a **formula** or **identity.**
- **Slide 23:** Ask delegates to read through the story, and work groups of 2/3 to identify which equations describe the story. (*See T-6 in instructions for further details*)

Individual work: writing equations: slide 24 *(See T-7)*

- Give each delegate *Card set: Stories* (not cut up)
- Ask spend 5 minutes writing an equation for each of the stories (*see T-9*)

Collaborative activity: matching cards: slide 25 *(see T7-8)*

- Arrange in groups of 2 or 3 and provide each group with:
    - Card set: Stories (cut up)

  - Card set: Equations
- Ask delegates to match each story with an equation
- As delegates finish, ask them to jot down the matched pairs on their mini-whiteboards
- Ask groups to compare their answers and reconcile any differences through discussion.
- Hold a brief plenary discussion.

Collaborative activity: posters - slide 26 *(see T8-9) (NB: optional - if time allows)*

- Remain in same groups of 3 and provide each group with:
  - Flipchart sheet and pens
  - Card Set: Steps to Solving
- Ask to divide flipchart sheet into 4 quadrants, and stick Equations E1-4 at the top of each quadrant (put other cards aside)
- Explain that they are now going to explore the steps to solving each equation, using the *Card set: Steps to Solving*. In between each step they should write a description of the process used (e.g. 'divide both sides by 2'). Repeat this until a solution is reached.
- Delegates may find more than one method for an equation, in which case the two solutions should be stuck side-by-side.

Plenary *(see T9-10)*

- Select 2 or 3 delegates from different groups who have completed *E1* and/or *E3.* Ask them to explain why there are 2 methods for solving these equations.
  - Which of the two methods is the most efficient?
  - Which method do you prefer? Why?

Session 4: Developing proportional reasoning

(Adapted from: Mathematics Assessment Project http://map.mathshell.org)

Aims:

To help delegates to:

- Reflect on the range of contexts in which ratio and proportion are used
- Examine proportional problems and appreciate their multiplicative structure
- Distinguish between direct proportion and other functional relationships
- Solve proportionality problems by efficient methods

Resources:

- Mini-whiteboards, pens and erasers
- Mathematics Assessment Project: Classifying Proportion and Non-Proportion Situations (downloadable PDF document)
    - Full instructions
    - Card set: Direct Proportion or Not?
    - Card set: Swapping Questions

Timing: approx. 40 mins

Activity:

- **Slide 27:** Ask for explanations of what the symbolic form '5:2' means and brainstorm different contexts where ratios like this might apply – in GCSE maths and also in work, vocational - e.g. conversions (currencies, metric/imperial, etc), recipes, paint colours, betting odds, sand and cement, rates (e.g. speed, unit prices – anything with 'per'), enlargements, pie charts. (Also make links with fractions and percentages). Highlight that concepts and processes of ratio and proportion are key to many areas GCSE maths.

Properties of direct proportion: slides 28-29 *(see T5-6)*

- **Slide 28:** Explain that this problem has some numbers missing; ask delegates to suggest two reasonable numbers to put in, and to write these on their mini-whiteboards. Note down their ideas in a table on the board, eg:

Grams bought	100	1000	200	50	500
Total cost	£1.20	£12	£2.49	£0.60	£7.20

- Ask delegates to share their methods. (These will vary – including informal halving and adding strategies).

  Ask: *Which numbers on the board are easier to use? Why?*

  Now ask delegates to choose some harder numbers to use and write them on their mini-whiteboards, and add them to the table. Check for any mistakes and misconceptions.

- Draw attention to the properties of direct proportion through questioning:
  - *What are the two quantities (variables) in this situation?* (Grams bought, total cost)
  - *What do you notice about the relationship between them?*
  - *What is the cost of **one gram** of cheese?* (£0.012)
  - *How could you now find the cost for any number of grams in **one** step?* (Just multiply every amount by £0.012 to get the total cost.)

- Explain to students that the two variables are directly proportional to each other. Now ask students for ideas on the properties of direct proportion. Write their ideas on the board. Then ask the following questions in turn:
  - *How much would it cost if you buy zero ounces? (Zero pounds) What happens to the total cost if you double the amount you buy? (The cost doubles)*
  - *What would a graph of this situation look like? (Straight line through origin.) Make a sketch of the graph on your mini-whiteboards.*

- After a few minutes ask two or three students with contrasting graphs to explain them. Encourage the rest of the class to ask questions and challenge their reasoning.

- **Slide 29** summarizes the properties of direct proportion. Distribute this as HO3 *(Properties of direct proportion)* for reference on the next activity.

Collaborative group work: writing own questions: slide 30 *(T6-8)*

- Arrange in groups of 2 or 3 and provide each group with:
  - Card set: Direct proportion or not?

- Explain that each card contains 2 quantities, with blanks instead of numbers. Task is to put numbers into these blanks and then classify the cards based on whether or not the two quantities vary in direct proportion. Go through instructions on slide 30, checking understanding.

- Listen and watch delegates as they complete the task, noting their methods. (Do they check all three properties from slide 29 before classifying?) Try to support delegates' thinking rather than prompting them to use a particular method.
- Challenge delegates to use difficult numbers (with fractions or decimals) the second time they write on a card. Ask what methods they've used. Suggest that they try using the same method second time through.

Collaborative group work: sharing and answering questions: slide 31-33 *(T8-9)*

- Give each group a couple of blank *Swapping Questions* cards and ask students to pick a completed *Direct Proportion or Not?* card and copy it onto a blank *Swapping Questions* card.
- Give students a few minutes to copy out their questions, then ask them to exchange questions with a neighbouring group.
- As the neighbouring group finishes answering the questions, ask them to swap back and check each other's work. Show slide 33 to help explain to students what they have to do.
- After a few minutes, ask students to work again with their neighbour and discuss any differences in answers and methods used.

Plenary discussion: slide 34 *(T9-10)*

- Ask groups to highlight any cards that resulted in differences of opinion and allow them to explain how they resolved these differences. Ask other delegates to compare the methods described with those that they used.
- Ask delegates to explain whether each question is a direct proportion question, referring to the list of properties on HO3.
- **Slide 34:** discuss the questions on the slide.

Reflection (slide 35)

- Ask delegates to reflect on the activities carried out during sessions 3 and 4, in line with slide 35, and discuss in table groups. Invite them to make record their individual reflections in their handbooks.
- Take brief plenary feedback from each table.

Session 5: Exploring straight line graphs

Aims:

To help delegates to:

- Make connections between proportional situations, straight line graphs and linear equations.
- Interpret the general equation for straight line graphs: $y = mx + c$.
- Explore how varying a straight line equation affects the gradient and y-intercept.
- Relate straight line graphs and equations to everyday contexts.

Resources:

- Mini-whiteboards, pens and erasers
- Graph paper, rulers and pencils
- Computer with graphing application for (e.g. GeoGebra), or Internet access to the *Maths is Fun* website: https://www.mathsisfun.com/data/straight_line_graph.html
- Laptop computers or tablets (1 between 3) *(optional - but it is recommended that participants bring their own lap top or tablet with GeoGebra pre-loaded)*
- HO: Linear graphs task
- Card sets:
    - 8a. Equations
    - 8b. Graphs
    - 8c. Contexts
    - 8d. Tables *(extension)*

Timing: approx. 40 min.

Activity:

- **Slide 36:** Remind delegates of the 'buying' cheese' scenario from the previous activity. Ask to imagine a caterer buying cheese in bulk – how much would 5 kg cost? Provide graph paper, and ask them to work individually or in pairs to plot a line graph for weights of cheese between 0-5 kg. They should now use their graphs to answer the questions and display these on mini-whiteboards. Highlight answers will not be exactly the same – why is this?

- **Slide 37:** Now ask delegates to work in pairs to produce a simple formula to link the cost to the weight purchased (i.e. c = 6w), and display these on mini-whiteboards. Highlight that the 6 is a ***constant*** because it doesn't change, while c and w are ***variables*** because they change. They should now use their formula to answer the same two questions.
- **Slide 38:** Check whether answers were the same using graphical and algebraic methods. Briefly discuss reasons for discrepancies and the relative merits of the two methods.
- **Slide 39:** Similarly, for the mobile phone scenario, ask delegates to work in pairs to produce a simple formula to link the cost to the number of minutes used (i.e. c = 0.05m + 10), and display these on mini-whiteboards. Next, ask them to plot a graph of the same relationship, then answer the questions, checking their answers against their formula.
- **Slide 40:** Introduce the general formula for a straight line graph: **y = mx + c**, pointing out the constants and variables, and relating this to the mobile phone scenario. Use questioning to bring out how the cheese buying scenario fits in:
    - *What is the value of **m** for the cheese buying?* (6)
    - *What is the value of **c**?* (0)
    - *What does it tell you if the value of c is zero*? (The graph passes through the origin (0,0). The relationship is proportional)

Exploring straight line graphs using graphing software

- For this activity, you will need a graphing application for (e.g. GeoGebra). Alternatively, use the *Maths is Fun* website: https://www.mathsisfun.com/data/straight_line_graph.html Where possible, delegates should have the same app loaded on a laptop or tablet, so they can experiment independently.
- Ask the group what they think the formula y = x would look like and sketch these on min-whiteboards. Enter this into the app and ask to compare with their sketch.
- At this point, either carry on leading an exploration of straight line graphs from the front, following HO4 (*Plotting graphs of equations*) **OR** provide HO4 for delegates to explore the graphs independently in groups of 2 or 3 (if laptops/tablets are available).
- Afterwards ask participants:
    - *How could you determine the value of **m** just by looking at the graph?*
    - *How could you determine the value of **c**?*

Collaborative activity: multiple representations of equations

- Arrange in groups of 2 or 3 and provide each group with:
  - 8a. Card set: Equations
  - 8b. Card set: Graphs
- Work collaboratively to match the graph cards with the equation cards, making sure that delegates state their reasoning.
- Next, provide **8c. Card set: Contexts** to each group and match these to their equations and graphs.
- ***Extension for those who complete the task****:* Make further sets of 3 matching cards OR provide **8d. Card sets: Tables** to match with other sets.
- Take plenary feedback.

Session 6: Comparing different approaches

(Adapted from: Mathematics Assessment Project http://map.mathshell.org)

Aims:

To help delegates to

- Interpret a situation and represent the variables mathematically.
- Select appropriate mathematical methods to use.
- Interpret and evaluate the data generated and identify the break-even point, checking it for confirmation.

Resources:

- Mini-whiteboards, pens and erasers
- Full instructions from Mathematics Assessment Project: Comparing Value for Money: *Baseball Jerseys* (downloadable PDF document)
- HO8: Football Shirts
- HO9: Sample responses to discuss

Timing: approx. 30min.

Activity:

- **Slide 41:** Arrange in groups of 2 or 3 and provide each group with HO5 *Football Shirts*. Ask them to solve the problem in whatever way they wish; ask them to show their working in an organised way. As far as possible, allow delegates to answer the questions without assistance.
- **Slide 42:** After students have had sufficient time to discuss some different approaches, distribute copies of the *Sample Responses to Discuss* to each group. Ask groups to compare the different approaches – and how they are similar of different to the approaches they used. Ask them to annotate each piece of work, indicating any mistakes made and how the work could be improved.
- **Slide 43:** Take plenary feedback in line with the questions on slide.

- Finish by highlighting that learners have started to look at simultaneous equations in this activity.

Reflection (slide 44)

- Ask delegates to reflect on the activities carried out during sessions 1 and 2, in line with slide 44, and discuss in table groups. Invite them to make record their individual reflections in their handbooks.
- Take brief plenary feedback from each table.

Session 7: Sequences

Aims:

To help delegates to:

- Understand the concepts, processes and notation of arithmetic sequences.
- Relate arithmetic sequences to linear equations and straight line graphs.

Resources:

- Mini-whiteboards, pens and erasers
- HO6: Looking at structure

Timing: approx. 25min.

Activity:

- **Slide 45:** Discuss the linear equation shown, and ask what it would look like as a graph:
    - *What would be its gradient?*
    - *Where would it cross the y-axis?*

  Ask delegates to work in pairs to complete the table of value to go with the equation. Take feedback and write the table on the board.

- **Slide 46:** Display the table of values, and ask the accompanying questions. Highlight that what they have is a linear sequence – also known as an 'arithmetic sequence' – with term-to-term difference of 2.

- **Slide 47:** Discuss how the sequence can be represented more visually – as with the *numicon* blocks in the picture.
    - *Can you see how the blocks relate to the first 3 terms in the sequence?*
    - *Can you see how the blocks relate to the original equation?* (y = 2x + 3)
    - *Where can you see the 2x in the blocks? What does x relate to?*
    - *Can you see the 3 in the blocks?*

- **Slide 48:** Reveal the sequence one term a time, asking delegates to predict what the next line will say. Finally, reveal the n^{th} term, and discuss what this means. Explain that for sequences we use 'n' instead of x, y, etc. This can be used to calculate any term in the sequence. Ask if they can you see the 2 and 3 from the original equation.

- **Slide 49:** Show the first sequence, and ask delegates to write the term-to-term difference on their mini-whiteboards. Repeat for an expression for the n^{th} term. Discuss any mistakes or misconceptions. Repeat for the 2^{nd} sequence.
- **Slide 50:** Introduce the activity with reference to the first sequence. Provide HO6 *(Looking at structure)* and ask to work in groups of 2 or 3 to complete.
- Take brief plenary feedback:
    - What did you like about the activity?
    - What have you learnt about arithmetic sequences?
    - How could you adapt this approach for your learners?

Session 8: Exploring algebra exam questions

Aims:

To enable delegates to:

- Explore the demand of GCSE exam questions with regard to ratio, proportion and algebra.
- Work collaboratively to apply maths skills in solving problems.

Resources:

- Selection of ratio, proportion and algebra questions from specimen papers for the new GCSE maths examinations (2016) – foundation tier.
- Flipchart sheets and markers

Timing: approx. 25 mins.

Activity:

- **Slide 51:** Arrange in groups of 2 or 3 and provide each group with a selection of ratio, proportion and algebra questions from the new (2016) GCSE exams. Ask delegates to skim through the questions and highlight suitable ones that they would like to work on – these should be challenging to them, but not beyond their reach.
- As learners attempt each question, they should paste the questions on a flipchart sheet, then work collaboratively to answer the question. Groups should ensure that all members of the group are involved and included, and members should challenge anything they disagree with or don't understand. Groups should annotate their posters with their working out and reasoning.
- Completed questions should be displayed on the wall of the room. After delegates have had the opportunity to answer a few questions, suggest that they have a look at other group's posters and see if they can follow their methods and solutions.
- **Slide 52:** Finish with a short plenary discussion about working on exam questions. What are the advantages and disadvantages of working collaboratively? Would you do this with your learners? What other ideas do you have for working on exam questions?

Review and Evaluation

- **Slide 53:** Review what has been covered during the day, and ask delegates to reflect on what they have got out of today, and to note down:
    - One thing I have learnt today
    - One thing I will do as a result of today
- Ask a few volunteers to read out their responses
- Slide 54: Remind to complete evaluation forms before departing