CONCEPT DEVELOPMENT

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

Classifying Proportion and Non-proportion Situations

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley

Classifying Proportion and Non-Proportion Situations

MATHEMATICAL GOALS

This lesson unit is intended to help you assess whether students are able to:

- Identify when two quantities vary in direct proportion to each other.
- Distinguish between direct proportion and other functional relationships.
- Solve proportionality problems using efficient methods.

COMMON CORE STATE STANDARDS

This lesson relates to the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

7.RP: Analyze proportional relationships and use them to solve real-world and mathematical problems.

This lesson also relates to the following *Standards for Mathematical Practice* in the *Common Core State Standards for Mathematics*, with a particular emphasis on Practices 2, 5, 6, and 8:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

INTRODUCTION

This lesson unit is structured in the following way:

- Before the lesson, students work individually on a task designed to reveal their current levels of understanding. You review their solutions and write questions to help them improve their work.
- At the beginning of the lesson, there is a whole-class discussion about key features of direct proportionality.
- Students then work in small groups on a task related to the assessment task. They write and solve their own questions on direct proportion, exchange questions with another group, assess each other's work, and write suggestions for improvement.
- In a whole-class discussion, students share their questions and solution methods, generalizing to identify criteria for identifying direct proportion.
- In a follow-up lesson, students use their learning and your questions to review their work.

MATERIALS REQUIRED

- Each individual student will need a calculator, a mini-whiteboard, a pen, an eraser, a copy of the assessment task, *Getting Things in Proportion,* and a copy of the review task, *Getting Things in Proportion (revisited)*.
- Each small group of students will need a copy of the lesson tasks, *Direct Proportion or Not?* and *Swapping Questions*.

TIME NEEDED

15 minutes before the lesson, a 60-minute lesson, and 15 minutes in a follow-up lesson. These timings are approximate: exact timings will depend on the needs of your class.

BEFORE THE LESSON

Assessment task: *Getting Things in Proportion* (15 minutes)

Give this task, in class or for homework, a few days before the formative assessment lesson. This will give you an opportunity to assess the work and to find out the kinds of difficulties students have with it. You should then be able to target your help more effectively in the subsequent lesson.

Explain what you would like students to do:

> *Read this task carefully.*
>
> *Spend a few minutes answering the questions on the sheet. Make sure to explain all your reasoning carefully.*
>
> *Do not be too concerned if you cannot finish everything. [Tomorrow] we will have a lesson on these ideas, which should help you to make further progress.*

It is important that, as far as possible, students are allowed to answer the questions without assistance. Some students may find it difficult to get started: be aware that if you offer help too quickly, students will merely do what you say and will not think for themselves. If, after several minutes, students are still struggling, try to help them understand what is required.

When all students have made a reasonable attempt at the task, reassure them that they will have time to revisit and revise their solutions later.

> *Do not worry if you have struggled with completing this task. We will have a lesson [tomorrow] that should help you improve your work.*

Getting Things in Proportion

Q1. Leon

Leon has $40.

How many Mexican Pesos can Leon buy with his dollars?

Explain how you figure this out.

Exchange Rate
$1 US = 12 Mexican Pesos

Q2. Minna

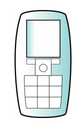
This is the call plan for Minna's cell phone:

$15 a month plus free texts plus $0.20 per minute of call time.

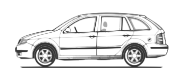
Minna made 30 minutes of calls this month, and 110 texts.

How much does she have to pay the phone company?

Explain how you figure this out.



Q3. Nuala



Nuala drives to her grandma's.

She drives at 20 miles per hour.

The journey takes 50 minutes.

How long would the journey take if Nuala drove at 40 miles per hour?

Explain how you figure this out.

Q4. Orhan

Orhan mixes some purple paint.

He uses three pints of blue paint for every five pints of red paint.

Orhan wants to mix more paint exactly the same color.

He has 17 ½ pints of red paint.

How much blue paint does he need?

Explain how you figure this out.

Q5. Here are two statements about the math in Q1 to Q4 above.

For each question, decide which statements are **true**. Explain your answers.

	If you double one quantity, you double the other.	**The ratio: *first quantity : second quantity* is always the same.**
Q1 Leon	Dollars, Mexican Pesos	Dollars : Mexican Pesos
Q2 Minna	Minutes, Dollars	Minutes : Dollars
Q3 Nuala	Speed, Time	Speed : Time
Q4 Orhan	Blue paint, Red paint	Blue paint : Red paint

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding and any difficulties they encounter.

We suggest that you do not score students' work. Research shows that this will be counter productive as it will encourage students to compare their scores and distract their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given in the *Common issues* table on the next page. These have been drawn from common difficulties observed in trials of this unit.

We suggest you make a list of your own questions, based on your students' work. We recommend you either:

- write one or two questions on each student's work, or
- give each student a printed version of your list of questions and highlight the questions for each individual student.

If you do not have time to do this, you could select a few questions that will be of help to the majority of students and write these on the board when you return the work to the students in the follow-up lesson.

Common issues	Suggested questions and prompts
Uses mental strategies For example: The student has calculated solutions, but written very little.	• Explain in more detail how you figured out your solution. • How can you make your mathematical reasoning clear to others?
Uses informal strategies For example: The student has used doubling and halving with addition.	• Can you think of a method that could be used for any quantity? E.g. What if you had ... cans of paint? • You had to do a lot of work to figure out that answer. Can you think of a more efficient way of solving this kind of problem?
Identifies the problem structure as additive rather than multiplicative when answering proportional questions For example: The student calculates 3 + 12½ (Q4).	• What is the relationship between red cans and blue cans of paint? • How many cans of blue paint would you use for one single red can? How can you use that in your solution?
Uses method of cross multiplying proportions when answering proportional questions For example: The student writes $\frac{17\frac{1}{2}}{x} = \frac{5}{3}$, which is correct, but manipulates the equation incorrectly (Q4). Or: The student writes: $\frac{x}{5} = \frac{17\frac{1}{2}}{3}$, which is incorrect (Q4).	• Which of these numbers relate to red paint? Blue paint? Explain how your method works.
Does not recognize when quantities vary in direct proportion For example: The student does not answer Q5. Or: The student says that the paint question (Q4) is not a proportional relationship. Or: The student claims that the cell phone question (Q2) is a proportional relationship.	• What properties must proportional relationships have? • Does this relationship have the following properties? - It is a linear function; - if one quantity is zero so is the other; - if one quantity doubles, so does the other.
Does not justify claims For example: The student distinguishes correctly between proportional and other functions, but does not explain how s/he made the distinctions.	• What properties do proportional relationships have? • How do you know this relationship is directly proportional?
Completes the task	• Write an interesting question from everyday life in which the quantities vary in direct proportion. Now answer your own question.

SUGGESTED LESSON OUTLINE

Whole-class introduction: *Properties of Direct Proportion* (10 minutes)

Students often incorrectly think that in proportional situations the figures in the problem influence the operation used in the calculation. One aim of this discussion is to address this misconception. Another aim is to check that students have vocabulary ('proportional', 'quantities that vary in direct proportion', and 'proportional relationship'*)* to talk about that structure.

Give each student a mini-whiteboard, a pen, and an eraser.

Show Slide P-1 of the projector resource:

Buying Cheese

10 ounces of cheese costs $2.40.

Ross wants to buy ……… ounces of cheese.

Ross will have to pay $ ....

This problem has some numbers missing.

Suggest two reasonable numbers to put in.

Ask students to show you their mini-whiteboards and note down their ideas in a table on the board:

Ounces bought	10	100	20	5	15
Total cost	$2.40	$24	$4.80	$1.20	$3.60

Ask students to share their methods. These will vary. Typically, some may use informal halving and adding strategies, such as:

10 ounces costs $2.40, so 5 ounces costs $1.20, so 15 ounces costs $2.40 + $1.20 = $3.60.

Are any of the numbers on the board easy to use? Why?

Now choose some harder numbers to use.

Again ask students to show you their whiteboards and add their ideas to the table:

Ounces bought	10	100	20	5	15	8	27	3½
Total cost	$2.40	$24	$4.80	$1.20	$3.60	$1.92	$6.48	$0.84

You may find some students think you must divide when small numbers are used and multiply when large numbers are used.

Mike, why did you choose these numbers?
How did you figure out the cost?

Does someone have a different method?

Which method do you prefer? Why?

So now we have a question that we could answer.

Draw students' attention to the properties of direct proportion.

What are the two quantities (variables) in this situation? [Ounces bought, total cost.]

What do you notice about the relationship between them?

How could you find the cost for any number of ounces in ***one*** *step?*
[One ounce costs 24 cents, so you could just multiply every amount by 0.24 to get the total cost.]

Explain to students that the two variables are directly proportional to each other.

Now ask students for ideas on the properties of direct proportion. Write their ideas on the board.

Then ask the following questions in turn:

How much would it cost if you buy zero ounces? [Zero dollars.]

What happens to the total cost if you double the amount you buy? [The cost doubles.]

What would a graph of this situation look like? [Straight line through origin.]
Make a sketch of the graph.

After a few minutes ask two or three students with contrasting graphs to explain them. Encourage the rest of the class to ask questions and challenge their reasoning.

Slide P-2 of the projector resources summarizes the properties of direct proportion:

Properties of Direct Proportion

- One quantity is a multiple of the other.
- If the first quantity is zero,
  the second quantity is zero.
- If you double one quantity,
  the other also doubles.
- The graph of the relationship
  is a straight line through the origin.

Leave the list of properties on the board during the lesson. It is important that students have this Slide to refer to during the subsequent parts of the lesson.

Collaborative small-group work (1): write your own questions (20 minutes)

Organize students into pairs or groups of three.

Give each group a pack of cards from the lesson task *Direct Proportion or Not?* and explain to students what they are being asked to do:

I've given you some cards relating to different situations.

There are two quantities in each situation, with blanks instead of numbers.

You are going to put numbers into these blanks and then classify the cards based on whether or not the two quantities vary in direct proportion.

Display Slide P-3, which shows the instructions for working:

Working Together

Choose one of the cards to work on together.

1. Choose some easy numbers to fill in the blanks.
   Answer the question you have written.
   Write all your reasoning on the card.
2. Now choose harder numbers to fill in the blanks.
   Answer your new question together.
   Write all your reasoning on the card.
3. Decide whether the quantities vary in direct proportion.
   Write your answer and your reasoning on the card.

When you have finished one card, choose another.

Go through these carefully and check that students understand what they are being asked to do.

While students write their questions you have two tasks: to note different student approaches to the task and to support student learning/problem solving.

Note different student approaches

By carefully listening and watching students as they work together you will get a better idea of students' range of understanding, be in a better position to ask questions to help them progress, and be more purposeful in who you select to explain solutions to the whole-class.

Notice the methods students use to solve the problems. Do they use multiplication or informal doubling and halving strategies? Do they use the same methods when they introduce harder numbers? Do students choose efficient methods? Can they use those methods to solve problems accurately? Do students check their solutions and try to make sense of the answers? Are students able to identify the properties of direct proportion? Do they check all three properties from Slide P-2 before classifying?

Support student learning/problem solving

Try to support students' thinking and reasoning, rather than prompting them to use any particular method. You may find the questions in the *Common issues* table useful.

If the whole-class is struggling on the same issue, you could write one or two relevant questions on the board and hold a brief whole-class discussion.

Challenge students to use difficult numbers (with fractions or decimals) the second time they write on a card. Ask what methods students have used. Suggest that students try using the same method second time through.

Is your method still effective?

Can you think of a method that will work, no matter how difficult the numbers?

Ask questions to help students notice the properties of proportional relationships that have already been noted.

Is the relationship between the amount of fuel and the cost directly proportional?
How do you know?

Draw students' attention to other properties of direct proportion as they emerge in their work.

What would the graph look like?

Can you change the numbers for this card (INTERNET or CELL PHONE) so that the quantities are directly proportional? [Put the fixed charge equal to zero.]

Prompt students to compare cards:

How are these two problems similar and how are they different?

If students are progressing well, you could ask them to sketch on the cards the graphs of each situation.

Collaborative small-group work (2): sharing questions and answers (15 minutes)

Give each group a couple of blank *Swapping Questions* cards.

Ask students to pick a completed *Direct Proportion or Not?* card and copy it onto a blank *Swapping Questions* card.

You are now going to exchange questions with another group.

You can pick the easy numbers version or the hard numbers version.
Make sure you do not write in the answer!

Choose one that you think is a direct proportion question and another that is not.

Give students a few minutes to copy out their questions.

Then ask them to exchange questions with a neighboring group.

Work together as a pair.

Answer the questions the other group has given you.

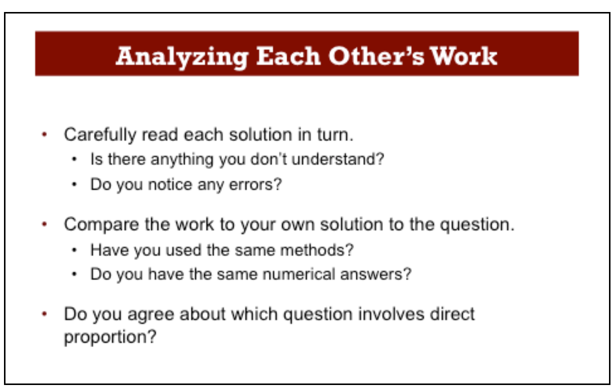
Write all your calculations and reasoning on the card.

Decide which question is a direct proportion question and which is not and write reasons for your decisions.

Give students time to work on this. Support them in recording all their calculations and reasons for decisions in writing.

As students are finishing, ask them to swap back and check each other's work.

Show Slide P-4 to help explain to students what they have to do:



After a few minutes, ask students to work again with their neighbor.

Take turns to discuss any differences you see.

It is important that you all agree and understand the methods used.

While students are working, observe and support as before. Notice whether students have any common errors or difficulties. It may be useful to discuss these as a whole-class. Note two or three questions for which there was disagreement for use in the whole-class discussion that follows.

Whole-class discussion (15 minutes)

You may find it helpful to use Slides P-5 to P-12 in the projector resource to support this discussion. These show the cards from the lesson task.

Begin by asking groups to choose a card that resulted in differences of opinion. Allow them to explain how they resolved these differences. Ask other students to compare the methods described with those they used on their own versions of the problem.

Which story situations were the easiest for you to solve? What made them so?

Which of the story situations were the most challenging to solve? What made them so?

When you found that you were 'stuck', what did you do to get 'unstuck'?

Did you solve your version of the question the same way?
Did anyone use a different method?

Ask students to decide whether the group's question is a proportion question. Ask them to explain why they agree or disagree with the decisions about direct proportionality, referring to the list of properties on Slide P-2.

Is your version of the question a proportion question? How do you know?
Does anyone agree? How do you know?
Did anyone decide differently? What properties did you not find?

Once students have reached an agreement about the classification of the card, encourage them to generalize.

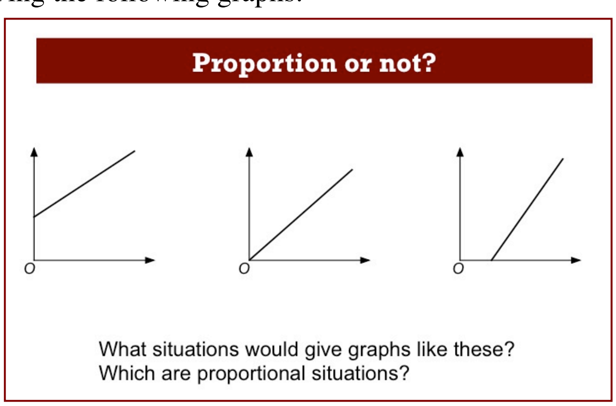
When you're deciding whether or not it's a proportion question, does it make any difference what numbers you put in? [It makes a difference to the cards INTERNET and CELL PHONE.]

Choose some numbers that make the relationship proportional. [Fixed cost = 0.]
How does the graph change? Show me. [Students may recognize that for the graph $y = mx + b$; only when $b = 0$ is the relationship directly proportional.]

Is there any difference between the graph representing the quantities for the INTERNET and the graph representing the quantities for the CELL PHONE? Show me.

Students may note, for example, that INTERNET is directly proportional if they ignore the free minutes. This corresponds to changing the 'zero' on the time axis. All linear relationships can be converted to directly proportional relationships by a suitable translation of either variable.

Project Slide P-13 showing the following graphs:



Describe situations that could be represented by each of these graphs.
Which are proportional and which are not?

Follow-up lesson: *Getting Things in Proportion (revisited)* (15 minutes)

Give students their scripts from the first task, *Getting Things in Proportion* and a copy of the new task, *Getting Things in Proportion (revisited)*.

If you have not written questions on individual students' scripts, display your list of questions on the board now. Students can select from this list those questions they think apply to their own work.

Do you recall the work about direct proportion? Remember how you wrote your own questions?

I would like you to spend some time reviewing your work. Read through your script and my questions carefully. Answer these questions and revise your response.

Now have a go at the new task, Getting Things in Proportion (revisited). Can you use what you've learned to help you to answer these new questions?

Some teachers like to give this task for homework.

SOLUTIONS

Assessment task: *Getting Things in Proportion*

Q1 This proportion question does not require a succinct or formal method and may elicit effective but inefficient use of repeated addition for multiplication, or strategies involving doubling and halving with addition. Two possible methods are shown below.

Mental doubling.

For each \$1 Leon could buy 12 Mexican Pesos.
Doubling, for \$2 he could buy 24 Mexican Pesos.
Doubling again, for \$4 he could buy 48 Mexican Pesos.
Multiplying by 10, for \$40 he could buy 480 Mexican Pesos.

Direct multiplication.

For each \$1, Leon could buy 12 Mexican Pesos.
For \$40, he could buy 40 times as much. $40 \times 12 = 480$ Mexican Pesos.

Q2 The cell phone plan is a linear but not a proportional relationship.
Minna must pay $\$15 + \$0.20x$, where x is the number of minutes per month. (Students may try to make use of the number of texts, even though they are free and so do not contribute to cost.) Minna pays $\$15 + \$0.20 \times 30 = \$15 + \$6 = \$21$.

Q3 Nuala takes 50 minutes when driving at 20 miles per hour. The distance is fixed. If she drives twice as fast, she will get there in half the time, in 25 minutes. (This is an 'inverse proportional relationship'.) Students sometimes use an inefficient method that involves calculating her journey distance. They might calculate $20 \times (50 \div 60)$ to find the distance in miles and then divide this answer by 40 to calculate the time in hours.

Q4 As with Q1, this question involves a proportional relationship and students may make use of various effective but sometimes inefficient methods.

Method A

For 5 pints of red paint Orhan needs 3 pints of blue.
For 5 + 5 + 5 pints of red, 3 + 3 + 3 pints of blue.
For 2 ½ pints of red, 1½ pints of blue.
So for 17½ pints of red paint, Orhan needs 10 ½ pints of blue paint.

Method B

The amount of red paint increases from 5 to 17½.

This gives a scale factor of $\frac{17\frac{1}{2}}{5} = \frac{35}{10} = 3.5$

The amount of blue paint required is $3.5 \times 3 = 10.5$ or 10½ pints.

Method C

The amount of red paint per pint of blue paint is $\frac{3}{5}$

So the amount of blue paint required for 17½ pints of red paint is

$$\frac{3}{5}\times 17\frac{1}{2}=\frac{3}{5}\times\frac{35}{2}=\frac{21}{2}=10\frac{1}{2}\text{ pints.}$$

Q5

	If you double one quantity, you double the other.	***first quantity : second quantity*** **This ratio is always the same.**
Q1 Leon	True. The number of Mexican Pesos is 12 × the number of dollars. If you double the number of dollars, you double the number of Mexican Pesos. Students may show this by providing a few examples.	Dollars : Mexican Pesos True. The ratio is Dollars : Pesos = 1 : 12
Q2 Minna	False. Minna pays \$15 + \$0.20x, where x is the number of minutes of calls. If x = 1, \$15 + \$0.20 = \$15.20 If x = 2, \$15 + \$0.20 × 2 = \$15.40. 15.40 ≠ 2 × \$15.20.	Minutes : Dollars False. If x =1, \$15 + \$0.20 = \$15.20 giving 1 : 15.20. If x =10, \$15 + \$2 = \$17 giving 1 : 1.7.
Q3 Nuala	False. Doubling the speed halves the time it takes to cover the fixed distance to Grandma's house.	Speed (mph) : Time (minutes) False. If the speed is 20mph, the ratio is 20 : 50 = 2 : 5, but if the speed is 40 mph the ratio is 40 : 25 = 8 : 5.
Q4 Orhan	True. The ratio between the two quantities is fixed, so multiplying one quantity by a number increases the other quantity by the same factor.	Blue paint (pints): Red paint (pints) True. The ratio is given as 3 : 5.

Assessment Task: *Getting Things in Proportion (revisited)*

Again, students' methods may vary considerably so these solutions are only indicative.

Q1 The cost of the taxi ride is not a proportional relationship. Cherie must pay \$4 + \$1.50x, where x is the number of miles. So the total cost is 4 + 1.5 × 7 = \$14.50.

Q2 This is a proportional relationship. 13 cards at \$0.80 per card is 13 × 0.8 = \$10.40.

As before, this proportional question does not require a succinct or formal method, and may elicit effective but inefficient use of repeated addition for multiplication, or strategies involving doubling and halving with addition. For example, students may calculate

10 × 0.8 = 8 and then add on 0.8 + 0.8 + 0.8 = 2.4 giving a total of \$10.40.

Q3 In a scale drawing, the ratio between the length on the drawing and the real-life length is fixed. In this case, the width of the drawing of the room is 4"; the real room is 10' wide.

So the ratio is 4" : 10' or 1" : 2.5'. So 12.5' in real life is 5" on the plan.

Students may make a common error on ratio problems and see the relationship as additive. For example, a student might write:

4" : 10'. 10' increases to 12.5'. Increase by 2.5. The line on the plan should be 6.5" long.

Q4 This is not a proportional relationship. The time it takes to rise to the 5th floor is 40 seconds, but the lift then stops for 1 minute. After that, the lift continues to rise at a steady rate. In total, the lift takes 15 × 10 + 60 = 210 seconds to reach the 16th floor. Note that a common error here is to multiply by 16 rather than 15.

Q5

	If you double one quantity, you double the other.	***first quantity: second quantity*** **This ratio is always the same.**
Q1 Cherie	False. Cherie pays \$4 + \$1.50x, where x is the number of miles. If x = 1, \$4 + \$1.50 = \$5.50. If x = 2, \$4 + \$3 = \$7. Doubling the number of miles does not double the fare.	Distance (miles) : Cost (\$) False. The ratio changes as the distance varies.
Q2 Ellie	True. The number of cards × \$0.80 gives the price you pay. If you double the number of card you buy, you double the price you pay because this involves a fixed unit cost.	Number of cards : Cost (\$) True. The ratio is 1:0.8.
Q3 Dexter	True. Doubling the length on the plan doubles the length represented from the real room. If the line is 4" long, the real room is 10' long. If the line is 8" long, the real room is 20' long.	Length on drawing : length in room True. The ratio is 1:30.
Q4 Fred	False. The time taken to reach the fifth floor is 40 seconds. The time taken to reach the 10^{th} floor is 90 second plus an extra 60 seconds, that is, 150 seconds in total.	Floor the lift has reached : Time (seconds) False. The ratio changes as the lift ascends.

Lesson task: *Direct Proportion or Not?*

Driving The relationship between the variables is inverse proportion. Doubling the speed halves the time it takes to cover a fixed distance. In this case, $s = d/t$.

Cell Phone The relationship between the number of minutes and the cost may or may not be proportional, depending on the assumptions made. E.g. The relationship may be discrete, if fractions of minutes are rounded up to whole minutes in pricing.

If the cost per month is nonzero, the relationship will be linear, but not directly proportional. In that case, if the number of minutes of use in a month is zero, the phone will cost whatever the student specifies as the monthly charge.

If students decide that the cost per month is zero, the relationship will be one of direct proportion.

Internet The relationship between the GB used and the cost is directly proportional if zero GB are supplied free each month, otherwise the relationship is linear but not directly proportional.

Map The relationship between distances on the map and distances in real life is proportional.

Toast The relationship in 'Toast' involves a discrete function (the graph would be a step function). The toaster has two slots. Suppose it takes x minutes to toast two slices. If you make an even number of slices of toast, $2n$, it will take nx minutes to make all the toast.

Suppose you make $2n+1$ slices of toast ($n = 0, 1, 2...$). It will take as long to make the one extra slice as it would to make another two slices of toast. In that case, it will take $(n+1)x$ minutes to make all the slices.

number of slices	*1*	*2*	*3*	*4*	*5*	*6*	*7*
time	*x*	*x*	*2x*	*2x*	*3x*	*3x*	*4x*

Students may also want to consider how long it would take to make a fractional part of a slice of toast!

Smoothie The relationships between the ingredients will all be proportional.

Triangles In the card 'TRIANGLES', similarity is a directly proportional relationship. The ratio between corresponding sides **within** each triangle stays the same as the triangle is scaled. If one side were zero, the other side would also be zero, so the graph of one length against another would pass through (0, 0). The ratio of corresponding sides **between** two triangles is invariant. The numbers students put into the gaps will specify x.

Line Assume the straight line passes through the origin and the point (a, ka).
Then it will also pass through the point (x, kx) and the relationship between the coordinates is proportional.

Getting Things in Proportion

Q1. Leon

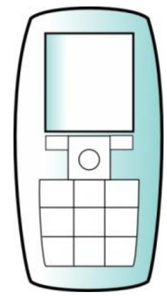
Exchange Rate $1 US = 12 Mexican Pesos

Leon has $40.

How many Mexican Pesos can Leon buy with his dollars?

Explain how you figure this out.

Q2. Minna



This is the call plan for Minna's cell phone:

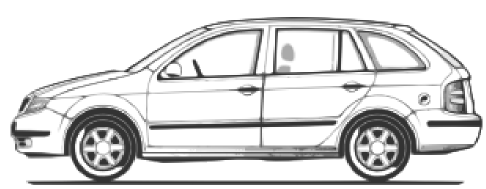
$15 a month plus free texts plus $0.20 per minute of call time.

Minna made 30 minutes of calls this month and 110 texts.

How much does she have to pay the phone company?

Explain how you figure this out.

Q3. **Nuala**



Nuala drives to her grandma's.

She drives at 20 miles per hour.

The journey takes 50 minutes.

How long would the journey take if Nuala drove at 40 miles per hour?

Explain how you figure this out.

Q4. Orhan

Orhan mixes some purple paint.

He uses three pints of blue paint for every five pints of red paint.

Orhan wants to mix more paint exactly the same color.

He has 17 ½ pints of red paint.

How much blue paint does he need?

Explain how you figure this out.

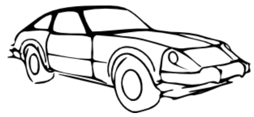
Q5. Here are two statements about the math in Q1 to Q4 above.

For each question, decide which statements are **true**. Explain your answers.

	If you double one quantity, you double the other.	**The ratio: *first quantity : second quantity* is always the same.**
Q1 Leon	Dollars, Mexican Pesos	Dollars : Mexican Pesos
Q2 Minna	Minutes, Dollars	Minutes : Dollars
Q3 Nuala	Speed, Time	Speed : Time
Q4 Orhan	Blue paint, Red paint	Blue paint : Red paint

Direct Proportion or Not? (1)

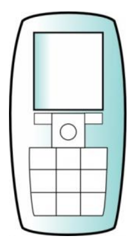
DRIVING



If I drive at ……... miles per hour, my journey will take ……... hours.

How long will my journey take if I drive at ……... miles per hour?

CELL PHONE

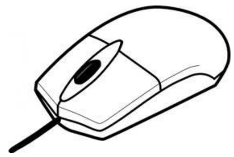


A cell phone company charges $......... per month
plus $.......... per call minute.

I used .... call minutes last month.
How much did this cost?

Direct Proportion or Not? (2)

INTERNET



An internet service provides .... GB
(gigabites) free each month.

Extra GB used is charged at $......... per GB.
I used .... GB last month. How much did this cost?

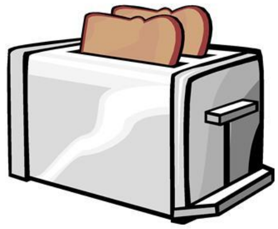
MAP

A road .... inches long on a map is .... miles long in real life.

A river is .... inches long on the map.
How long is the river in real life?

Direct Proportion or Not? (3)

TOAST



My toaster has two slots for bread.

It takes .... minutes to make .... slices of toast.

How long does it take to make .... slices of toast?

SMOOTHIE

To make three strawberry smoothies, you need:

.... cups of apple juice

.... bananas

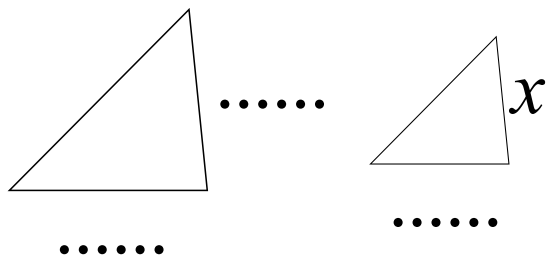
.... cups of strawberries

How many bananas are needed for .... smoothies?

Direct Proportion or Not? (4)

TRIANGLES

These triangles are similar.



Calculate the length marked x.

LINE

A straight line passes through the points (0, 0) and (…. , ….).

It also passes through the point (…. , y).

Calculate the value of y.

Swapping Questions

Getting Things in Proportion (revisited)

Q1. Cherie

Cherie wants to go home in a taxi.

She lives 7 miles away.

The taxi firm charges $4 plus $1.50 per mile.

How much will the fare be?

Explain how you figure this out.

Q2. Ellie

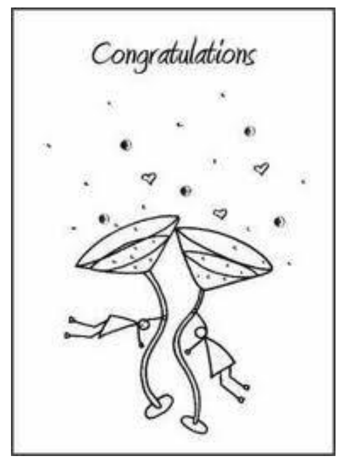
Ellie is buying greeting cards.

The cards cost $0.80 each.

She wants to buy 13 cards.

How much will she pay the store clerk?

Explain how you figure this out.



Q3. Dexter

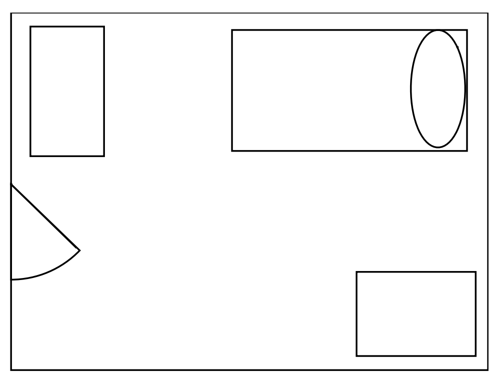
Dexter makes a scale drawing of his room.

In real life, the room is 10' wide and 12' 6" long.

In Dexter's drawing, the room is 4" wide.

What measure should the length be?

Explain how you figure this out.



Q4. Fred

Fred lives on the 16th floor.

The elevator goes up one floor each 10 seconds.

It stops at the fifth floor for 1 minute for people to get out.

How long does it take Fred to get to the 16th floor?

Explain how you figure this out.

Q5. Here are some statements about the math in Q1 to Q4 above.

For each question, decide which statements are **true**. Explain your answers.

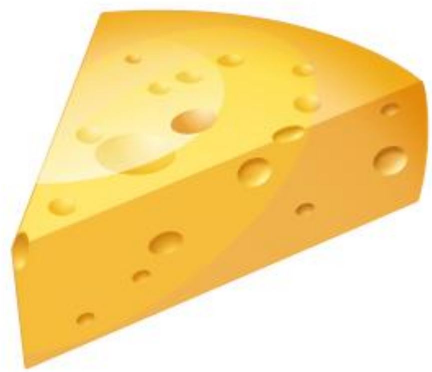
	If you double one quantity, you double the other.	**The ratio: first quantity : second quantity is always the same.**
Q1 Cherie	Distance, Cost	Distance : Cost
Q2 Ellie	Number of cards, Cost	Number of cards : Cost
Q3 Dexter	Length on drawing, Length in room	Length on drawing : Length in room
Q4 Fred	Time, Floor the lift has reached	Time : Floor the lift has reached

Buying Cheese

10 ounces of cheese costs $2.40.

Ross wants to buy ……… ounces of cheese.

Ross will have to pay $ ....



Properties of Direct Proportion

- One quantity is a multiple of the other.
- If the first quantity is zero,
  the second quantity is zero.
- If you double one quantity,
  the other also doubles.
- The graph of the relationship,
  is a straight line through the origin.

Working Together

Choose one of the cards to work on together.

1. Choose some easy numbers to fill in the blanks.
   Answer the question you have written.
   Write all your reasoning on the card.

2. Now choose harder numbers to fill in the blanks.
   Answer your new question together.
   Write all your reasoning on the card.

3. Decide whether the quantities vary in direct proportion.
   Write your answer and your reasoning on the card.

When you have finished one card, choose another.

Analyzing Each Other's Work

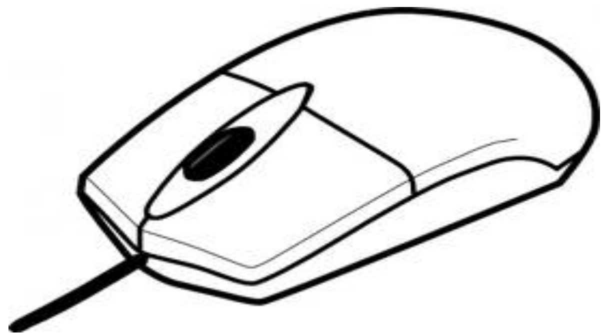
- Carefully read each solution in turn.
  - Is there anything you don't understand?
  - Do you notice any errors?
- Compare the work to your own solution to the question.
  - Have you used the same methods?
  - Do you have the same numerical answers?
- Do you agree about which question involves direct proportion?

Driving

If I drive at ……… miles per hour,
my journey will take ……… hours.

How long will my journey take
if I drive at ……… miles per hour?

Internet



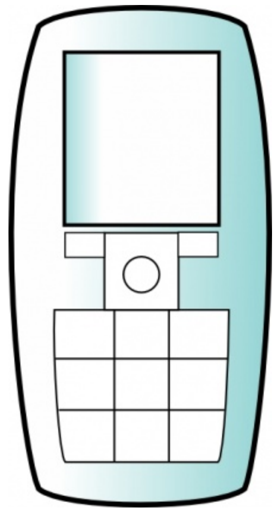
An internet service provides ……… GB (gigabites) free each month.

Extra GB used is charged at $ ……… per GB.

I used ……… GB last month.

How much did this cost?

Cell Phone



A cell phone company charges $……… per month plus $……… per call minute.

I used ……… call minutes last month.

How much did this cost?

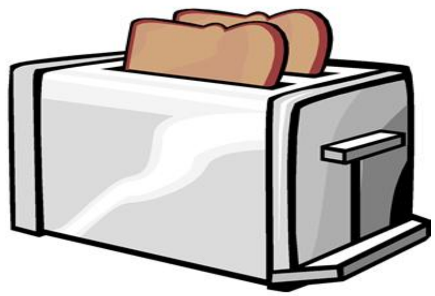
Map

A road ……… inches long on a map is ……… miles long in real life.

A river is ……… inches long on the map.

How long is the river in real life?

Toast



My toaster has two slots for bread.

It takes ……… minutes to make ……… slices of toast.

How long does it take to make ………slices of toast?

Smoothie

To make three strawberry smoothies, you need:

……… cups of apple juice

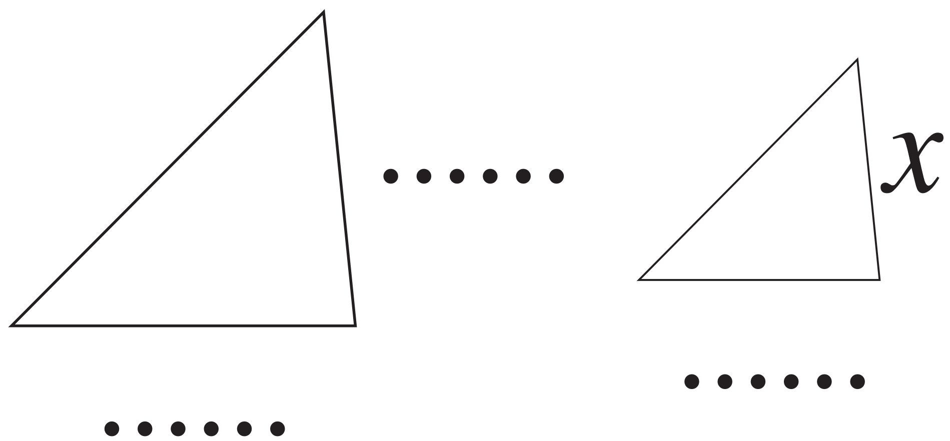
……… bananas

……… cups of strawberries

How many bananas are needed for …. smoothies?

Triangles

These triangles are similar.



Calculate the length marked x.

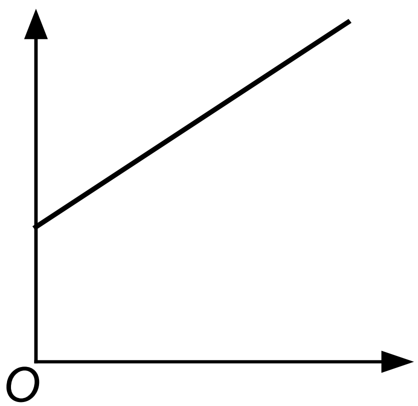
Line

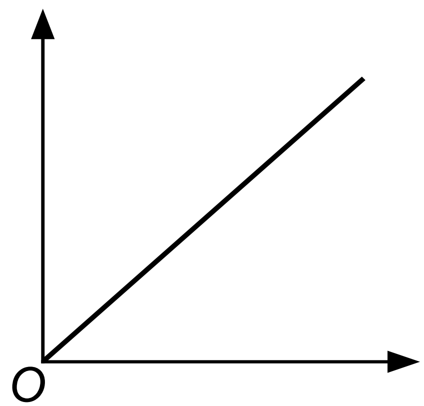
A straight line passes through the points (0,0) and (….. , …..).

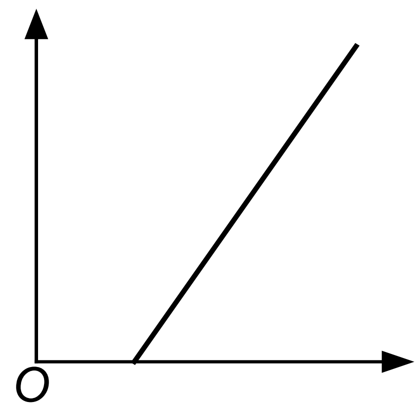
It also passes through the point (….. , y).

Calculate the value of y.

Proportion or not?







What situations would give graphs like these?
Which are proportional situations?

Mathematics Assessment Project

CLASSROOM CHALLENGES

This lesson was designed and developed by the
Shell Center Team
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Malcolm Swan, Clare Dawson, Sheila Evans,
Colin Foster and Marie Joubert
with
Hugh Burkhardt, Rita Crust, Andy Noyes, and Daniel Pead

It was refined on the basis of reports from teams of observers led by
David Foster, Mary Bouck, and Diane Schaefer
based on their observation of trials in US classrooms
along with comments from teachers and other users.

This project was conceived and directed for
MARS: Mathematics Assessment Resource Service
by

Alan Schoenfeld, Hugh Burkhardt, Daniel Pead, and Malcolm Swan
and based at the University of California, Berkeley

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