

Challenging topics

Understanding the laws of arithmetic: order of operations

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Introduction

This session will be useful to learners in a range of vocational and subject areas. It will be of particular interest to engineers; some of the examples are set in an Engineering science setting but examples from other vocational areas can be used instead. Most learners will already have used the laws of arithmetic but may not have mastered the concept that operations are completed in a particular order. They may have been taught a rule like BODMAS or BIDMAS but it is not enough to have learned the rule by rote, as the introduction to the session will show. They may still be uncertain about whether brackets are needed in expressions and where they have to be inserted to produce particular answers. This session and session 6b are intended to help learners to understand the underlying principles.

Learning objectives and outcomes

To help learners to:

- interpret numerical expressions using words and area representations
- recognise the order of operations
- recognise equivalent expressions.

Resources required

For each learner you will need:

- mini whiteboard and pen
- calculator.

For each small group of learners you will need:

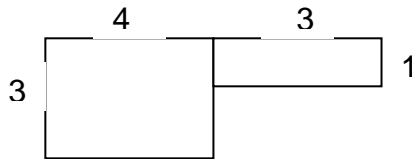
- Card set A: Calculations
- Card set B: Areas
- Card set C: Solutions
- the sheets for the 'Brackets' activity in Stage 5 (optional)
- felt tip pens (optional).

Starting points

The session assumes that learners are familiar with the use of indices or powers. They will also need to know how to calculate the area of simple shapes made from rectangles joined together.

You may need to check, through questioning, that learners understand these ideas. Examples may be:

- What is the difference between 3×2 and 3^2 ?
- Draw two different rectangles each with an area of 36.
- What is the area of this shape?



Planning learning in multiple environments

As teachers*, we spend a lot of time planning the learning experience in our classroom, workshop or training area. However, it is worth considering how we can encourage our learners to explore their learning in a wider range of settings. Pointing learners in the direction of a relevant TV programme, a newspaper article, or even an advertisement, can bring learning alive. There may be opportunities to ask learners to observe something on the way to work or college, to reflect on an aspect of their own social or home lives, or to apply a newly learned skill in a workplace situation. All these can help the learning process, making it more real and relevant.

Mathematics occurs everywhere in our lives; helping learners to recognise the occasions where it pops up, and to incorporate these into their own thinking, is a step towards real and useful learning.

* We use the word ‘teacher’ as a generic term to include teachers, tutors, trainers, lecturers and instructors in the further education (FE) system.

Time needed for this session

One hour.

Suggested approach

Stage 1: Beginning the session

Remind the group that engineers often use algebra to help them understand problems. Some examples might include:

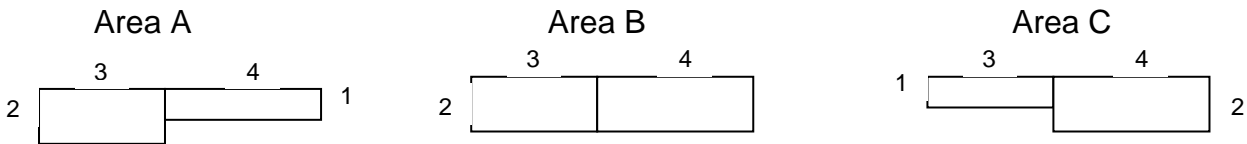
- Ohm’s law, to understand the relationship between current and voltage in an electrical component.
- The equations of motion, to help predict velocity, acceleration and position for moving objects.
- The equation for thermal expansion.

You may like to add some examples that the group has been working on in recent weeks.

When manipulating algebraic expressions, we need to understand how the expressions work and how they relate to one another. Explain that this session will help learners to understand these relationships.

Start with some simple examples about area, but explain that what they will learn can be applied much more widely.

Draw the following three shapes on the board and ask the following questions to probe learners' existing understanding. Ask them to show their answers on their mini-whiteboards.



- If you work out $3 + 4 \times 2$, which area are you working out? Explain how you know.
- If you work out $(3 + 4) \times 2$, which area are you working out? How do you know?
- What answers does your calculator give for these questions?
- Can you give me an expression for the other area?
- What is the difference between $(2 + 3)^2$ and $2^2 + 3^2$?
- Can you show me a diagram to explain the difference?

If learners struggle with any of these questions, explain that you will leave it for now and return to it later in the session.

Stage 2

Ask learners if they have heard of BIDMAS (or BODMAS) and ask them to explain what it means.

Explain the danger of using such a rule without understanding it.

For example, write the following on the board and ask the group to tell you where you have gone wrong and what the correct answer is.

$3 \times (3+1)^2 \div 2 - 5 + 9$	
$= 3 \times (4)^2 \div 2 - 5 + 9$	(Brackets)
$= 3 \times 16 \div 2 - 5 + 9$	(Index or power)
$= 3 \times 8 - 5 + 9$	(Division)
$= 24 - 5 + 9$	(Multiplication)
$= 24 - 14$	(Addition)
$= 10$	(Subtraction)

The mistake is at the 'Addition' step. When this is corrected, the answer is 28.

Stage 3: Working in groups

Arrange learners in pairs or groups of three.

Give each group **Card set A: Calculations**, **Card set B: Areas** and **Card set C: Solutions**.

Ask learners to place the cards face up on the table and take it in turns to match them in pairs or threes. When they find a pair or set of cards that belong together, they should place them face up and side-by-side, so that they are all visible. (They should not stack the cards as this makes it impossible for you to monitor work as you go round the room.)

Each time a learner matches two or three cards, they should try to explain to their partner(s) why the cards belong together. Encourage learners to challenge their partner(s) if they think an explanation is not clear enough.

Learners will soon realise that there are more Calculations cards than Areas and Solutions cards – do not comment at this stage.

Learners will find that some areas can be obtained by more than one calculation and that they need to provide additional solutions. They can use the blank cards for this.

Encourage learners to explain how they can immediately see when a Calculations card matches an Areas card, without working out answers. Ask them to look for alternative ways of finding the areas.

Learners who struggle with this activity could cut the compound shapes into rectangles and find the area of each rectangle before finding the area of the whole shape.

Learners who match the cards quickly may be challenged to move towards generalisation. Some sample questions are:

- What happens when we change the numbers on the cards?
- Suppose we change the 4 on every card to a 5? Will the Calculations cards still match in the same way?
- Will this still be true when we change the 4 to a large number, a negative number or a decimal?
- Do the Area pictures in Card set B help to explain why this happens?

Stage 4: Reviewing the learning

When learners have completed their matching, return to the questions asked in Stage 1.

What answers can the learners now give?

Using mini-whiteboards and whole group questioning, begin to generalise the learning:

- Draw an area that requires this calculation: $3 \times (4 + 5)$.
- Write a different calculation that gives the same area.
- Draw an area that requires this calculation: $\frac{6 + 8}{2}$
- Write a different calculation that gives the same area.
- Draw an area that requires this calculation: $(10 + 5)^2$.
- Write a different calculation that gives the same area.

Draw out the general learning points that have emerged:

- The equivalence of multiplying by $\frac{1}{2}$ and dividing by 2
- The order of operations:
 - a) brackets first
 - b) then powers or roots
 - c) then multiplication or division
 - d) then addition or subtraction.

Ask the group to draw on their mini-whiteboards an area representation (like the diagrams in **Card set B: Areas**) of the following engineering expressions.

$$u + at$$

$$\frac{1}{2}(u + v)t$$

$$u^2 + 2as$$

$$\frac{1}{2}h(a+b)$$

Stage 5: Extending the learning

Adding brackets

This activity can be used to consolidate what has been learned.

Arrange learners into teams of three or four. Give each team a felt tip pen and two copies of Sheet A, four copies of Sheet B and four copies of Sheet C. These can be A5 size.

Sheet A
 $2 + 3 \times 4 + 5$

Sheet B
 $2 \times 3 + 4 \times 5$

Sheet C
 $2 + 3 \times 4^2$

Call out a sheet name and a target number. Teams have to show how they can reach this target by adding brackets to the sheet. For example, if you call out 'Sheet A, 25', the first team to show you Sheet A with brackets in the following position will gain a point.

Sheet A
 $(2 + 3) \times 4 + 5$

Here are some answers, but learners may come up with others.

Sheet	Target	Method
A	29	$2 + 3 \times (4 + 5)$
A	45	$(2 + 3) \times (4 + 5)$
B	26	$(2 \times 3) + (4 \times 5)$ or no brackets
B	46	$2 \times (3 + 4 \times 5)$
B	50	$(2 \times 3 + 4) \times 5$
B	70	$2 \times (3 + 4) \times 5$
C	80	$(2 + 3) \times 4^2$
C	146	$2 + (3 \times 4)^2$
C	196	$(2 + 3 \times 4)^2$
C	400	$((2 + 3) \times 4)^2$

Further ideas

This activity uses multiple representations to deepen understanding of number operations. This type of activity can be used in any topic where a range of representations is used.

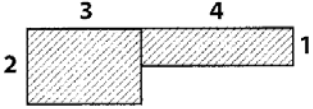

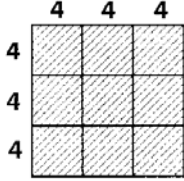


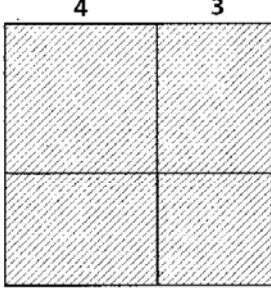

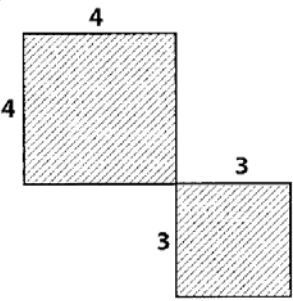
Examples in the Improving learning in Mathematics pack include:

- SS6 Representing 3D shapes
- S4 Understanding mean, median, mode and range.

Card set A – Calculations

A1 $3^2 + 4^2$	A2 $2 \times (3 + 4)$
A3 $(3 + 4)^2$	A4 3×4^2
A5 $(3 \times 4)^2$	A6 $\frac{3}{2} + \frac{4}{2}$
A7 $2 \times 3 + 4$	A8 $4 + 3 \times 2$
A9 $3^2 \times 4^2$	A10 $2 \times 3 + 2 \times 4$
A11 $\frac{1}{2} (3 + 4)$	A12 $3^2 + 4^2 + 2 \times 3 \times 4$
A13 $\frac{3 + 4}{2}$	A14 $\frac{3}{2} + 4$

Card set B – Areas

<p>B1</p> 	<p>B2</p> 
<p>B3</p> 	<p>B4</p> 
<p>B5</p> 	<p>B6</p> 
<p>B7</p> 	<p>B8</p> 

Card set C – Solutions

144	48
49	3.5
5.5	14